MASTER'S DEGREE THESIS

# MSc in Statistics and Operations Research

**Title: Optimal Supply Chain Strategy through Stochastic Programming**


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# Optimal Supply Chain Strategy through Stochastic Programming

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# Abstract

In this project, a new two-stage stochastic programming decision model has been developed to assess: (a) the convenience of introducing 3D printing into any generic manufacturing process, both single and multi-product; and (b) the optimal degree of postponement known as the customer order decoupling point (CODP) while also assuming uncertainty in demand for multiple markets. To this end, we propose the formulation of a generic supply chain through an oriented graph that represents all the deployable alternative technologies. These are defined through a set of operations for manufacturing, assembly and distribution, each of which is characterized by a lead time and cost parameters. Based on this graph, we develop a mixed integer two-stage stochastic program that finds the optimal manufacturing technology to meet the demand of each market, the optimal production quantity for each operation, and the optimal CODP for each technology. The results obtained from several case studies in real manufacturing companies are presented and analyzed. The work presented in this master's thesis is part of an ongoing research project between UPC and Accenture.

Keywords: Supply chain strategy, Postponement, Stochastic Programming, 3D Printing

# 1.Introduction

The master's thesis presented here is the result of the work done during a scholarship at Accenture Innovation Center for Supply Chains, where I was a member of the work team for a project in collaboration between Accenture and the Universitat Politècnica de Catalunya (UPC). The aim of this project was to study the advantages of ultra-postponement with 3D printing using the analytical tools of operations research. To guarantee a better understanding of the topic of this thesis, some concepts and definitions should be explained first.

### Supply Chain

A **supply chain** consists of all stages that are directly or indirectly involved in fulfilling a customer request. The supply chain not only includes the manufacturer and suppliers, but also transporters, warehouses, retailers, and the customers themselves (Chopra & Meindl, 2006).

Boone, Craighead and Joe (2007) state that there are as many techniques and strategies in **supply chain management** as there are disciplines that generate such strategies, as well as the customers they seek to serve. There is but one common goal of these supply chain strategies and techniques. They all seek cost reduction while at the same time they work to improve supply chain performance, customer satisfaction and the fulfillment of customer. In this work, we focus on postponement and speculation concepts as primary references for designing efficient strategies.

### Postponement

Van Hoek (2001) defines **postponement** as an organizational concept whereby some of the activities in the supply chain are not performed until customer orders are received, as opposed to **speculation**. In postponement, companies can then finalize the output in accordance with customer preferences and even customize their products. Meanwhile, they can avoid building up inventories of finished goods in anticipation of future orders. Moreover, transportation between warehouses and factories can be avoided by shipping products directly to the customer rather than keeping them in stock. It should be noted that this may lead to smaller sized shipments over longer distances. As a result, postponement is often more relevant when products are more sensitive to inventory than transport costs (e.g., higher value-added products with greater product variety).

Bowersox and Closs (1996) define three postponement paradigms:

- **Time postponement** delays the forward movement of goods until customer orders are received (delaying the determination of time utility).
- **Place postponement** is the storage of goods at central locations in the channel until customer orders are received (delaying the determination of place utility).
- **Form postponement** delays product finalization until customer orders are received (delaying the determination of form/function utility).

### 3D Printing

3D printing is a technology that uses an additive process for manufacturing three-dimensional objects from a digital model. This manufacturing technology uses a computerized design file to generate successive layers of the desired material. Rather than cutting away raw material or using molds, as is oftentimes the case in traditional manufacturing, it is thus additive rather than subtractive or formative. While there are many different specific technologies and materials used in 3D printing, the main operating principles are the same.

## Framework: the Accenture TechLabs – BarcelonaTech Research Project

As a part of the Accenture Open Innovation Initiative, Accenture Tech Labs awarded 11 research grants to top universities around the world in order to significantly broaden and deepen the relationships between leading university researchers and Accenture's technology research and development groups. The project "**Digitalizing Supply Chain Strategy with 3D Printing**", proposed by the UPC, was one of 11 projects awarded.

This research project aims to study the advantages of ultra-postponement with 3D printing by using analytical tools and mathematical optimization models. The project was developed from July 2015 to July 2016 by the Group on Numerical Optimization and Modeling (GNOM-UPC) and the "Fundació CIM-UPC" (FCIM) in collaboration with the Accenture Analytics Innovation Center for Supply Chains (Barcelona) and Accenture Technology Labs (San José, California).

The original statement of the problem, as it was posed to the UPC team by Accenture TechLabs, had two commitments:

- From the point of view of 3D printing technology, to research and analyze the **different types of product categories that 3D printing can address** today. That was the main concern of the FCIM team within the project.
- From the point of view of operations research, to **identify, quantify, and analyze supply chain strategies** that would be used based on the manufactured item. The question addressed by Accenture TechLabs was literally "***When should an enterprise***

***implement certain strategic supply chain model that use 3D printing****?*". That was the main involvement of the GNOM team on the project.

## Objectives

The overall objective of this work was to find an answer to the question "*When should an enterprise implement certain strategic supply chain models that use 3D printing?*" by using the tools of operations research, specifically mathematical optimization models (probably stochastic). Due to the GNOM team's lack of experience in both 3D printing and supply chain problems, and also keeping in mind the wide scope of the question and the novelty of the topic (new 3D printing technologies in manufacturing), there were few *a priory* assumptions about how to tackle the problem. Thus, the foreseen practical objectives of this thesis were:

1. To perform an extensive review of the existing bibliography on analytical methods for evaluating supply chain strategies (speculation/postponement) and manufacturing with 3D printing.
2. To gather several test cases from Accenture's clients and FCIM, specifically those in which 3D printing can be an alternative manufacturing technology for the supply chain.
3. To formulate a mathematical optimization model for evaluating how to introduce 3D printing in the supply chain.
4. To obtain a computational implementation of the mathematical optimization model with some mathematical modeling language.
5. To use the mathematical model to assess how to introduce 3D printing and postponement in the supply strategy for some test cases provided by Accenture.

We will describe in this document how each one of these objectives has been fulfilled.

## Contribution

The main contributions of this thesis to the existing literature are:

1. A flexible network configuration of a generic supply chain that allows formulating a wide range of supply chain problems and features, such as: process selection, postponement degree and location.
2. An original stochastic programming model that contraposes the first stage against the recourse variables in an experimental and conceptual way; thus, it is able to decide the best technology to use from among a set of possible choices, as well as the optimal degree of postponement for each one of the selected technologies.

3. A novel treatment of uncertainty in the demand, specifically one that allows taking into account an approximation of the customer's waiting time without needing to model either a multi-period program or an explicit queuing system.

## Contents

In Section 2, we describe all the documentation we reviewed on qualitative and quantitative approaches to supply chain management, postponement and 3D printing. We also summarize some ideas and methods for modeling the problem.

Section 3 defines the problem and it is where we characterize a generic supply chain, demand and time periods, with all their associated parameters. Later, in Section 4, the defined problem is modeled through families of variables, constraints and cost terms.

Section 5 deals with the computational implementation of this problem as well as data generation. Test cases from real enterprises are described and solved in Section 6, showing different optimal solution in terms of delivery parameters.

Finally, Section 7 summarizes some general conclusions of the model, of the project and it discusses future research.

## Acknowledgments

I would like to thank Professor Javier Heredia for including me in this project, Professor Pedro Delicado for his dedication to the probability issues of the model, Professor Mari Paz Linares for being so strong, and also the rest of the team: Asier and Joaquim of Fundació CIM and Robert, Kiron, Ping, Mary, Sunny and Elsa of Accenture.

*"Las decisiones se toman en el momento de tomarse" – M.R.*

# 2.State of the Art

In this section, we first go over all the documentation from the literature that we reviewed, in particular that from which we draw concepts and ideas about supply chain management and mathematical programming. Then, we discuss common factors in quantitative works and applied methods, specifically in terms of their objectives. Finally, we summarize the key points in order to form the basis of describing and formulating our project.

## Reviewed documents

The purpose of the bibliographical research was:

1. To establish the conceptual background of supply chain strategies.
2. To study the existing works that relate manufacturing to 3D printing.
3. To envisage the mathematical methodology for analyzing the problem.

First, we focused on qualitative documents in order to delve into the main concepts and the research done not only on supply chain and postponement, but also on 3D printing. Then, we reviewed the problems raised and the developed approaches in order to analyze those factors that we must include in our project.

### Qualitative Analysis

The next table shows these qualitative documents that we have reviewed for general information on supply chain management.

| | Author | Year | Title | Journal |
|---|---|---|---|---|
| **1** | (van Hoek) | 2001 | The rediscovery of postponement a literature review and directions for research | Journal of Operations Management |
| **2** | (Brun & Zorzini) | 2009 | Evaluation of product customization strategies through modularization and postponement | International Journal of production economics |
| **3** | (Kemal) | 2010 | Postponement in Retailing Industry: A case study of SIBA | Master's Thesis on International Logistics and Supply Chain Management |

Table 1: Qualitative documents reviewed

## 3D Printing Research

| | Author | Year | Title | Journal |
|---|---|---|---|---|
| **1** | (Berman) | 2012 | 3-D printing: The new industrial revolution | Business Horizons |
| **2** | (Nyman & Sarlin) | 2014 | From Bits to Atoms: 3D Printing in the Context of Supply Chain Strategies | 2014 47th Hawaii International Conference on System Science |
| **3** | (Bhasin & Bodla) | 2015 | Impact of 3D Printing on Global Supply Chains by 2020 | MIT Master's Thesis |

Table 2: 3D printing documents reviewed

## Quantitative Models in Postponement

The following table shows the list of works reviewed for this project, in particular those that proposed some quantitative methods for coping with postponement. Bold references correspond to the most relevant contributions for our work.

| | Author | Year | Title | Journal |
|---|---|---|---|---|
| **1** | (Lee, Billington, & Carter) | 1993 | Hewlett-Packard gains control of inventory and service through design for localization | Interfaces |
| **2** | (Lee & Tang) | 1997 | Modeling the costs and benefits of delayed product differentiation | Management Science |
| **3** | (Ernst & Kamrad) | 2000 | Evaluation of supply chain structures through modularization and postponement | European Journal of Operational Research |
| **4** | (Aviv & Fedegruen) | 2001 | Capacitated Multi-Item inventory systems with random and seasonally fluctuating demands: Implication for postponement strategies | Management Science |
| **5** | (Bish & Wang) | 2004 | Optimal investment strategies for flexible resources, considering pricing and correlated demands | Operations Research |
| **6** | (Hsu & Wang) | 2004 | Dynamic programming for delayed product differentiation | European Journal of Operational Research |

| | | | | |
|---|---|---|---|---|
| **7** | (Chod & Rudi) | 2005 | Resource flexibility with responsive pricing | Operations Research |
| **8** | (Biller, Muriel, & Zhang) | 2006 | Impact of price postponement on capacity and flexibility investment decisions | Production and Operations Management |
| **9** | (Goyal & Netessine) | 2007 | Strategic technology choice and capacity investment under demand uncertainty | Management Science |
| **10** | (Bish, Lin, & Hong) | 2008 | Allocation of flexible and indivisible resources with decision postponement and demand learning | European Journal of Operational Research |
| **11** | (Lus & Muriel) | 2009 | Measuring the impact of increased product substitution on pricing and capacity decisions under linear demand models | Production and Operations Management |
| **12** | (Bish & Suwandechochai) | 2010 | Optimal capacity for substitutable products under operational postponement | European Journal of Operational Research |
| **13** | (Ngniatedema, Fono, & Mbondo) | 2015 | A delayed product customization cost model with supplier delivery performance | European Journal of Operational Research |

Table 3: Quantitative documents reviewed

## Qualitative Analysis of Supply Chain Strategies

In this section, we summarize the most relevant concepts that support the different supply chain strategies found in the literature. Special attention has been paid to the description of postponement and its benefits, together with some other related topics such as differentiation, customer order decoupling point and modularization.

### Supply Chain Strategies

When facing the strategic design of a supply chain, authors identify two opposite alternatives: **speculation** and **postponement**.

Regarding speculation, Buckin (1965) says:

> *"The principle of **speculation** holds that changes in form, and the movement of goods to forward inventories, should be made at the earliest possible time in the marketing flow in order to reduce the costs of the marketing system"*

With respect to postponement, Alderson (1957) established that:

*"[T]he most general method which can be applied in promoting the efficiency of a marketing system is the **postponement** of differentiation . . . postpone changes in form and identity to the latest possible point in the marketing flow; postpone change in inventory location to the latest possible point in time".*

Other relevant concepts for supply chain strategies that are closely related to postponement are:

- **Mass Customization**: the ability to supply products and services customized to suit individual customer specifications through high agility, flexibility and integration (Davis 1987).
- **Modularization**: a product design approach whereby the product is assembled from a set of standardized constituent units (Ernst & Kamrad, 2000).
- **Customer Order Decoupling Point**: the point which separates the forecast-driven production from the order-driven production in a flow of goods (Wikner & Rudberg, 2005).

In the following sub-sections, we are going to pay more attention to how different authors have considered and analyzed the concept of postponement.

### Postponement

Postponement refers to delaying any decision in the supply chain as much as possible until the placement of an order by a customer. Therefore, there can be several types of postponement, depending on the stage of the supply chain affected by the delay. Authors usually consider three different types of postponement, namely: **time**, **place** and **form** postponement:

| Types of postponement | Time postponement | Place postponement | Form postponement |
|---|---|---|---|
| **(Bowersox & Closs, 1996)** | Delaying the forward movement of goods until customer orders are received (delaying the determination of time utility) | Storage of goods at central locations in the channel until customer orders are received (delaying the determination of place utility) | Delaying product finalization until customer orders are received (delaying the determination of form/function utility) |

| | | | |
|---|---|---|---|
| **(Hoek, 1998)** | Delaying those activities that do not determine the forms and function of the products until orders are received | Delaying the movement of goods downstream in the chain until orders are received, thus centrally keeping goods and not making them place specific | Delaying those activities that determine the form and function of products until orders are received |

Table 4: Types of postponement

Kemal (2010) identifies some situations that can motivate the application of postponement:

| | |
|---|---|
| **Changing behaviors of consumers:** | Changes in consumer behavior are forcing retailers to move to postponement from speculation. Forecasting of consumer demand has been rendered difficult for the electronic retailers, due to ever-changing consumer behaviors. |
| **Uncertainties in Demand:** | Turbulences in markets and changes in technologies and consumer behaviors cause uncertainty in customer demands, which make forecasting difficult. |
| **Shortening Life Cycles of Products:** | Changes in demand conditions coupled with new technological developments have shortened the product life cycles. |
| **Increasing varieties of products:** | Technological changes and changes in behaviors of consumers have led to an increase in the variety of products offered. |
| **Mass Customization:** | The processes have become more complex due to the above-mentioned rapid changes in technology. |

This evolution in production can be beneficial to a postponed strategy and provide some main opportunities that Kemal (2010) enumerates, such as:

| Main Postponement Opportunities in Operations | |
|---|---|
| **Uncertainties** | Reduce risk of mixing volume and variety by delaying finalization of products |
| **Volume** | Make batches of one job |
| **Variety** | Presume and customize (requires flexibility) |
| **Lead Times** | Offer accurate response, yet perform activities within order cycle time |
| **Supply Chain Approach** | Reduce complexity in operations, yet possibly add flexibility and transport costs |

Table 5: Postponement opportunities

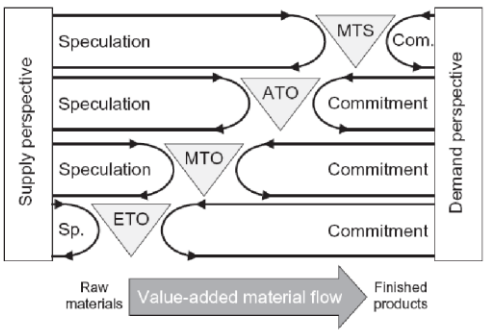
Finally, van Hoek (1998) summarizes:

- Some **key elements of the postponement concept** are used to characterize and classify existing work. These elements are:
    - Type of postponement and level of application in supply chain
    - Amount of customization
    - Spatial configuration of the chain
    - The role of operating circumstances
    - The role of change management

## Decoupling Point

Wikner and Rudberg (2005) define the **customer order decoupling point** (CODP), or simply decoupling point, as the point which separates the forecast-driven production (speculation) from the order-driven production (postponement) in a flow of goods. The CODP specifies the position in the chain where the customization occurs. Furthermore, the CODP indicates the extent to which operations are postponed and which are speculated.

Wortmann and Timmermans (1997) propose a CODP typology that depends on the position of the CODP in the supply chain stages (engineering, making, assembly, stock):




1. **MTS**: make-to-stock
2. **ATO**: assemble-to-order
3. **MTO**: make-to-order
4. **ETO**: engineer-to-order

Figure 1: Decoupling Point typologies (Kemal, 2010)

where ETO corresponds to the highest degree of postponement (pure postponement) and MTS to the lowest degree (pure speculation).

Generalizing the categorization of Wortmann and Timmermans (1997) in a given general supply chain process (i.e., a sequence of precedent actions or operations, from the acquisition of raw materials to the selling of a finished product): the decision to position a decoupling point in one operation defines a **speculation/postponement strategy** that is characterized by first a speculative flow of goods from the initial point to the CODP, and then a postponed flow from the CODP to the assigned markets.

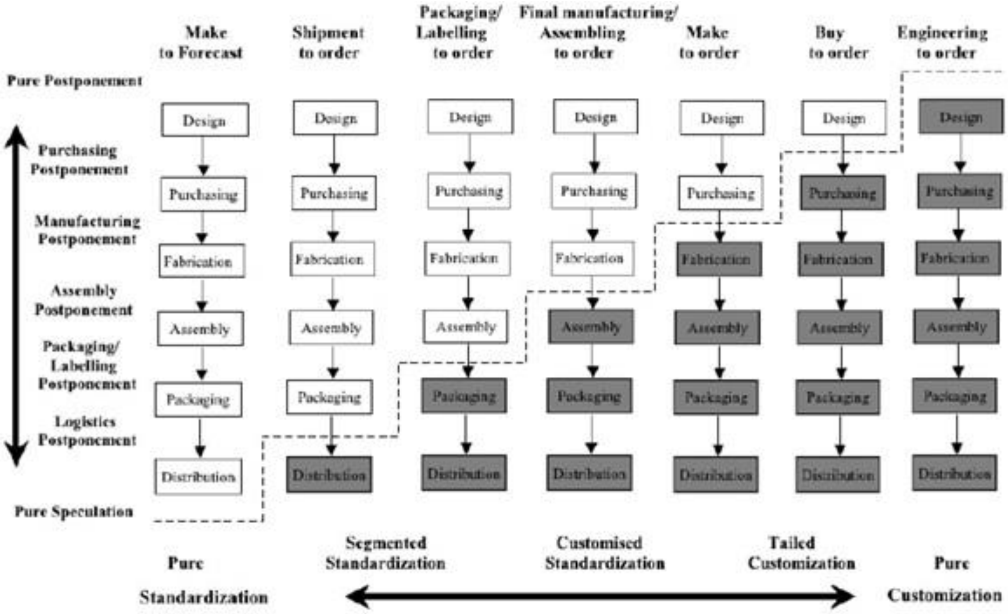



Figure 2: Decoupling point cases (Kemal, 2010)

Yang, Burns and Backhouse (2004) show in the table below how a speculation/postponement strategy can be in a given supply chain, specifically in terms of the decoupling point decision. It passes through mixed strategies as it runs from pure speculation to pure postponement. Of course, the key issue here is how to find the optimal placement of the CODP for a specific company. Some qualitative advice can be found in the bibliography. For instance, Brun and Zorzini (2009) suggest that the relevant factors influencing the optimal positioning of the CODP can be divided into three categories:

| | |
|---|---|
| **Product characteristics:** | Includes modular product design, customization opportunities, BOM (bill of material) profile and product structure complexity. |
| **Market-related factors:** | Refers to delivery lead-time requirements, demand volatility, demand volume, product range, product customization requirements, customer order size and frequency, and seasonality of demand. |
| **Manufacturing characteristics:** | Includes manufacturing lead time, number of planning points, flexibility, bottleneck position and sequence-dependent set-up times. |

Table 6: Factors influencing postponement

## Modularization

Starr (1965) introduced the concept of modularization in the literature. It implies a product design approach whereby the product is assembled from a set of standardized constituent units. Modular design effectively marries flexibility (of the end product) with standardization (of constituent parts).

Brun and Zorzini (2009) state that when modules are designed to allow component modification, modularity is used in the design and fabrication stages of the production cycle; while in the later stages (i.e., assemble and use), modules are added or interchanged but not altered.

Some authors study postponement and modularization as two concepts that are linked in the supply chain design. In this way, Brun and Zorzini (2009) state:

> *"The fundamental principle in these two concepts is essentially the same – marrying the advantages of scale of scope. While **modularization does this from a product design point of view, postponement attains it from a process design perspective**. Modularization essentially characterizes supplier responsibilities."*

Ernst and Kamrad (2000) qualitatively differentiate modularization as an inbound logistics concept and postponement as outbound manufacturing, doing so in the following way:

- **Inbound modularization** is the dimension that captures the degree of outsourcing and the usage of subcontractors for making the components.
    - **Low inbound modularization** represents a supply chain with a high degree of vertical integration.
    - **Highly inbound modularization**, however, is a highly decentralized supply chain that outsources many of the components from multiple suppliers.
- **Outbound postponement** captures the extent of customization offered in the supply chain. Therefore:
    - **High outbound postponement** is a supply chain basically organized around a make-to-order environment, where customer demand triggers the completion of the final product.
    - **Low outbound postponement** is characteristic of a make-to-stock environment, where an inventory of finished products is maintained in order to satisfy customer demand.

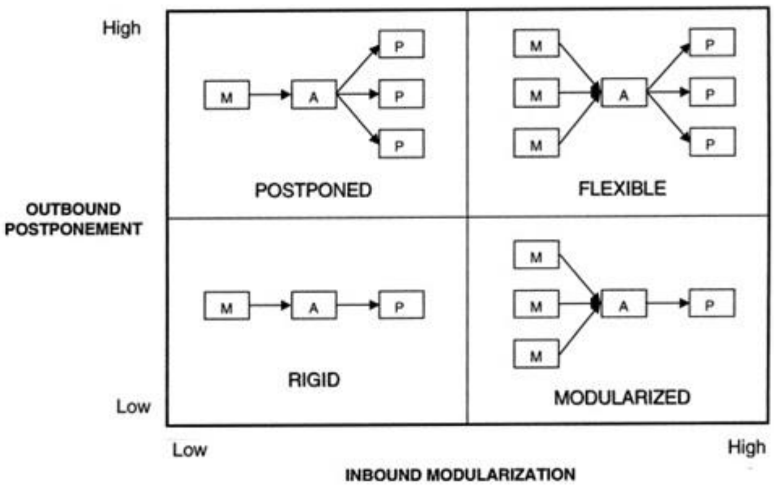



Figure 3: Postponement & modularization (Ernst & Kamrad, 2000)

The concept of modularization therefore appears to be more sensible for designing families of products with common parts than for quantitative problems of production and logistics. We will consider henceforth this concept as an opportunity for product design when supply chain strategy may need it, but we will not qualitatively discuss modularization for manufacturing operations.

## Differentiation Point

Some works center their analysis on form postponement, i.e., finding the optimal point (called the differentiation point) in the supply chain when semi-finished production becomes differentiated products. Models reviewed in this area often focus more on implementing some modular design in the supply chain than on other factors such as place or time postponement.

In these works we find a new concept that distinguishes two kinds of manufacturing operations that will be useful in later characterization of 3D printing technologies:

- **Flexible technologies**: those manufacturing operations that are able to produce different pieces or products with a single set-up, such as 3D printers or a craftsman.
- **Dedicated technologies**: those manufacturing operations that are designed to do a specific piece or process and unable to work with different pieces, such as a plastic injection operation.

In some of these studies, decisions about positioning the differentiation point in a given operation depends on the cost differential between when the given operation is common (for all products) or differentiated (for some specific product) versus the demand uncertainty cost (due to backordering and stock levels). It is at this point when the paradigm of flexible/dedicated technologies becomes relevant.

In our case, we will consider 3D printing technologies as flexible, and will quantitatively distinguish them from classic technologies (generally, dedicated) by assuming that flexible technologies are automatically amortized once they are purchased. This makes sense since a 3D printing machine can be used for further batches after the current one is over, for the lifetime of the machine.

### 3D Printing Characterization

There are few documents about 3D printing research in supply chain management, and none of those we reviewed include postponement or mathematically specific modeling. As with other manufacturing operations, we will characterize these technologies by their set-up costs along with the unitary production cost and lead time of a specific product. These parameters have been computed by Fundació CIM for each presented study case.

## Qualitative Models in Postponement

One possible classification of the reviewed documents distinguishes between works that propose a single-stage model and those that include a two-stage decision model. The next two tables show a summary of the most important characteristics of these models.

| Single-stage models | | | | | | |
|---|---|---|---|---|---|---|
| | **Postponement** | **# of manufac-turing steps** $N$ | **# of pro-ducts** $J$ | **Stochasticity** | **Decision variables (single)** | **Mathematical modeling (unevaluated minimization problem)** |
| **Ernst 2000** | Outbound postponement (extent of customization, form differentiation) | $N = 3$ (Manuf., Assembly, Packaging) | $J = 2$ | Demand $D_j \sim N(\mu_j, \sigma_j^2)$ Assumed known | Production quantity $S$ | $\min_S TC_i(S) \quad i \in \{R, F, P, M\}$<br>• $TC_i(S)$: total operational cost |
| **Lee 1997** | Form postponement (product differentiation) | $N > 1$ | $J = 2$ | Demand $D_j \sim N(\mu_j, \sigma_j^2)$ Assumed known | Differen-tiation point $n$ | $\min_{0 \le k \le N} Z(n) = I(n) + P(n) + ITI(n) + BI(n)$<br>• $I(n)$: total average investment cost.<br>• $P(n)$: total processing cost.<br>• $ITI(n)$: in-transit inventory cost.<br>• $BI(n)$: buffer inventory cost. |
| **Ngniatedema 2015** | Form postponement (product differentiation) | $N > 1$ | $J = 2$ | Demand $D_j \sim N(\mu_j, \sigma_j^2)$ Assumed known | Differen-tiation point $n$ | $\min_{0 \le n \le N} C(n) = Z(n) + Y(n) + G_0$<br>• $Y(n)$: average penalty cost resulting from supplier delivery time.<br>• $G_0$ : buffer inventory from the supplier side at the beginning of the production process. |

Table 7: Single-stage models reviewed

| Two-stage models | | | | | | | |
|---|---|---|---|---|---|---|---|
| | # of manuf. steps $N$ | # of products $J$ | Stochasticity | # of decision stages $K$ | First stage decision variables | Second-stage decision variables (postponed) | Mathematical modeling |
| **Biller 2006** | $N=1$ | Any $J>1$ | **Stochastic demand functions** $D_j = \epsilon_j - \alpha_j P_j$, $j = 1, \dots, J$ $\alpha_j$:calculated elasticity. $\epsilon_j$:stochastic demand intercept with probability scenarios $S$ | $K = 2$ | $K_j^D$ : dedicated plant capacity product $j$ $K^F$: flexible plant capacity. | $P_{js}$: price. $Q_{js}^D$: quantity using $K_j^D$. $Q_{js}^F$: quantity using $K^F$. | Two-stage stochastic programming model (numerical solution) |
| **Bish 2010** | $N=1$ | $J=2$ | **Inverse stochastic demand functions:** $P_j = \epsilon_j - D_j - \gamma D_{3-j}$, $j = 1,2$ $\gamma$: measure of product substitutability. $\epsilon_j$:stochastic price intercept, with joint known p.d.f. $h(\epsilon_1, \epsilon_2)$ | $K = 2$ | $K^F$ : flexible plant capacity. | $P_j(\epsilon_1, \epsilon_2)$: price. $Q_j(\epsilon_1, \epsilon_2)$: quantity prod. $j$ | Two-stage stochastic programming model (theoretical solution) |

Table 8: Two-stage models reviewed

Let us now analyze with more detail the characteristics of these models.

## Characterization of Demand

Ernst and Kamrad (2000), Ngniatedema et al (2015), Bish and Suwandechochai (2010) and many other authors have pooled the uncertainty inherent in supply chain problems into demand. Meanwhile, some other authors model this uncertainty as a probability distribution function (commonly normal) for analytically solving some problems. Another common approach is to develop two-stage stochastic models in order to differentiate a strategic 1$^{st}$ stage (design of supply chain and speculative production) from an operational 2$^{nd}$ stage (demand realization and postponed production).

Moreover, some authors model a decreasing linear relationship between demand and other variables, as Biller et al (2006) do with price. Also, some quantitative works study the correlation between two products by classifying them into substitutable products (negative correlation), complementary (positive correlation) or independent (no significant correlation).

## Characterization of Costs

The next characterization of costs – taken from Ernst and Kamrad (2000), Lee and Tang (1997) and others – is considered repeatedly in many works, and it consists of four main areas.

| Cost | Factor | Description |
|---|---|---|
| **Set-Up of operation** | Production set-up costs | Fixed cost associated with starting a manufacturing operation |
| **Production manufacturing** | Variable production costs | Cost of manufacturing one unit of production in an operation |
| **Backordering** | Variable stock-out costs | Cost of not selling a demanded unit not produced |
| **Stock** | Variable holding costs | Cost of stocking a unit of production |

Table 9: Cost characterization

Ernst and Kamrad (2000) develop an analytical formulation of costs in terms of (1) the probability distribution function of demand $f(x)$ and (2) production quantity $S$.

$$TC(S) = F + V(S) + H\int_0^S (S-x)f(x)dx + B\int_0^\infty (x-S)f(x)dx$$

This is similar to what Lee and Tang (1997) and Ngniatedema et al (2015) do.

## Capacity-Production-Price Paradigm

Bish and Suwandechochai (2010) and Biller et al (2006) study the impact of postponement through modeling stochastic programs, and they do so by taking decisions about production capacity (1$^{st}$ stage) as well as production quantity and selling price (2$^{nd}$ stage). These works do not contemplate geographical configuration or time variables. However, they do introduce another derivate strategy of postponement: price postponement.

These works illustrate a useful link between postponement and stochastic programming, with mention also of flexible and dedicated operations.

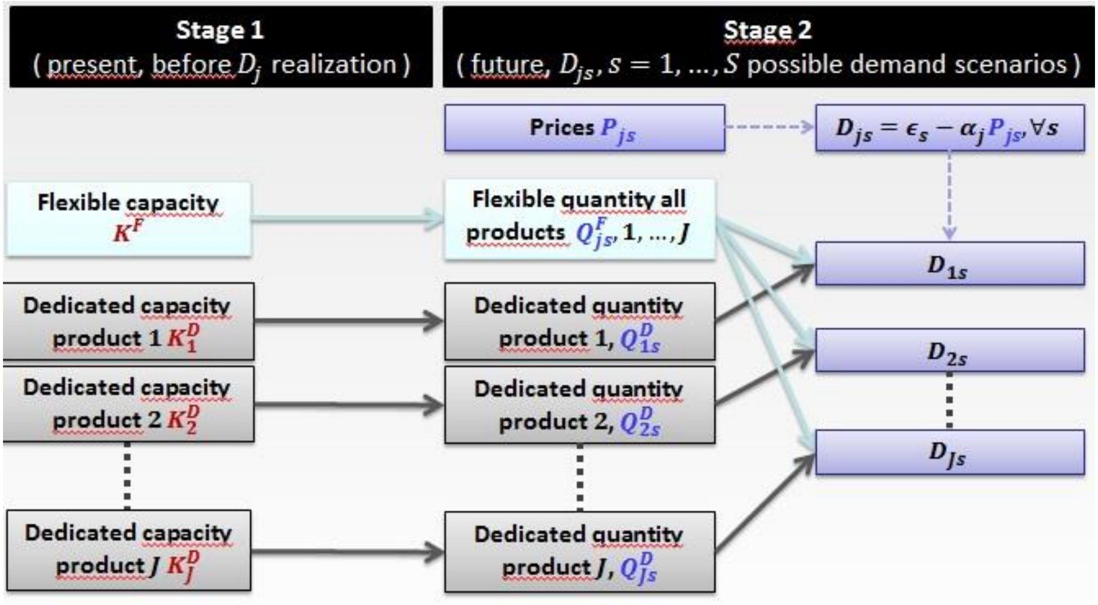



Figure 4: Capacity-production-price model

## n-Step Models

The works of Lee and Tang (1997) and Ngniatedema et al (2015) develop methods to find the optimal differentiation point in a supply chain that is composed of a sequence of $n$ steps, each including a supply chain operation and the possibility of installing a buffer. This configuration introduces a total cost expression by means of a cost structure similar to the one presented before, with the addition of some basic results from stock management. This cost expression is evaluated at any feasible point, i.e., at each operation of $n$.

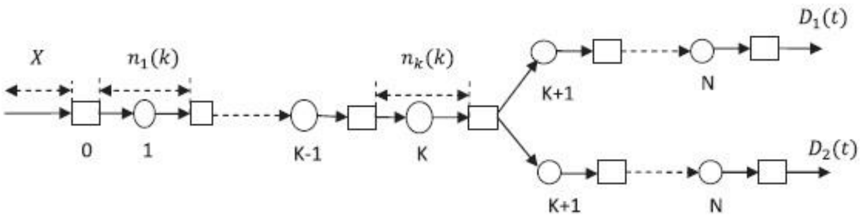



Figure 5: n-step model (Ngniatedema, Fono, & Mbondo, 2015)

These models are flexible insofar as they allow arbitrary n-step chains, but they do not offer advanced resolution methods or allow implementing problems of a facility location nature.

### Time Constraints

An essential limitation of postponement is the production and distribution time, known as lead time. When it production is postponed, one main factor is the time it takes until the demand is served.

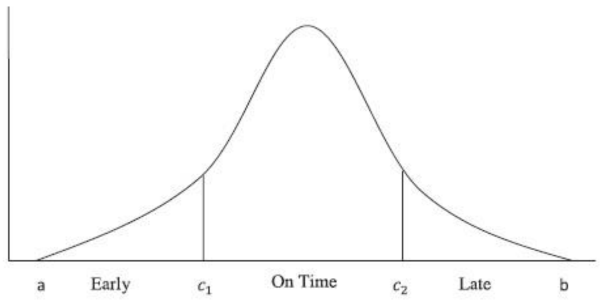



Figure 6: Delivery time (Ngniatedema, Fono, & Mbondo, 2015)

Ngniatedema et al (2015) introduce a system of temporal windows and costs for early and late deliveries, while other authors set a bound on maximum lead time.

## Proposed Modeling

Finally, we summarize a few points about the core idea we propose for developing a mathematical program to find the optimal degree of postponement.

### Stochastic Programming

A two-stage stochastic program can clearly describe both speculative (1st stage) and postponed (2nd stage) decisions. Speculative production has no time constraints, so it can go over more path flows than postponed production, which is finished after demand is realized so that production quantity may be bounded by a maximum delivery time.

Based on the works of Bish and Suwandechochai (2010) and Biller et al (2006), we will build a two-stage stochastic program to model the introduction of speculation/postponement strategies.

### Demand Uncertainty

As in most of the literature review, random variables of this stochastic program will be demand, which will be discretized in a finite number of scenarios from historic information, forecasting or the assumption of a given probability distribution function. Over a total demand of the time horizon studied, we will also decompose possible realizations of demand by time periods.

## Cost Characterization

Decision parameters for getting an optimal strategy will be mainly the ones analyzed previously, from the work of Ernst and Kamrad (2000) and others: those about set-up of operations, production costs, holding and stock-out costs.

## Time

This model will also consider time periods (but will not be a multi-period model) in order to introduce a maximum delivery time, as well as different realizations of demand by periods in the same scenarios. When postponement is selected, the feasible solution will be bounded to those cases when total lead time is below that maximum delivery time.

## Decoupling Point

A main factor that characterizes a speculation/postponement strategy is the point where a decoupling point is positioned, i.e., where speculative production is stored until demand is realized, in order to be finished and delivered in postponement.

These decisions, amply illustrated by Kemal (2010), condition which is studied as 1st or 2nd stage, so we must put conditions to allow our model to decide which operation shall be decoupling points, instead of solving a given number of cases.

## Network Configuration

A common limitation of many works is the scope of the chains designed. While some characterize a generic process as sequences of $n$ steps, others focus on specific configurations or directly exclude them.

We want the scope of the next model to be large and can adapt many real cases: as some objectives of this project are about the introduction of 3D printing technologies, we would consider process selection as well as strategy, production and stocks. Also, a flexible modeling of supply chains will allow us to include aspects of form and place postponement.

Therefore, we will design the so-called supply chain graph, a network consisting of nodes (supply chain operations) and arcs (precedence relations between operations) that create flow paths from initial nodes to markets.

## Conclusions

As a conclusion, the contribution of this work to speculative/postponement research will be the modeling of an optimization problem that (1) considers cost and time datasets as well as demand probability scenarios, (2) decides which operations are selected in the supply chain to manufacture some production, and (3) which manufacturing strategy is used in each operation and where the decoupling points are located.

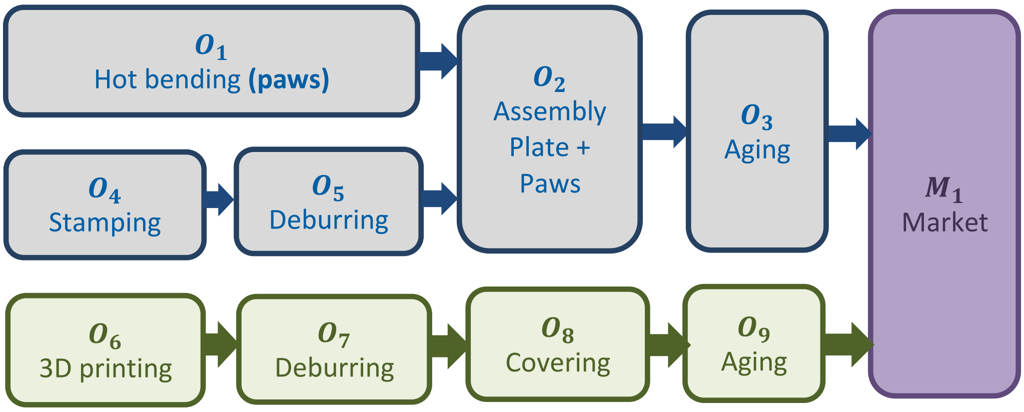
This contribution provides new tools for the field of supply chain management by offering a systemic analysis, as opposed to those methods focused in local issues, such as stock levels or sales. Also, the flexible network configuration of the supply chain allows the study of a wide range of problems related to time, place and form postponement, as well as those related to differentiation points, stock optimization, facility location problems…

# 3. Problem Definition

The Optimal Supply Chain Strategy ($OSCS$) problem is a two-stage stochastic MILP that, given a set of operations, determines **efficient processes** and **speculation/postponement strategies** for monopolist manufacturing. In this contribution, we propose the network modeling of a supply chain through an oriented graph that represents all the alternative technologies that can be deployed.

## Supply Chain Graph

All the possible supply chain configurations are represented through an oriented graph that contains all the alternative manufacturing processes. In this graph, the nodes correspond to the operations that define the processes (such as manufacturing, assembling, packaging or distribution), and the arcs represent the precedence between operations within a given process. An example of such a graph would be:




| Process | Operations |
|---|---|
| $P_1$ | $O_1, O_2, O_3, O_4, O_5 \rightarrow M_1$ |
| $P_2$ | $O_6, O_7, O_8, O_9 \rightarrow M_1$ |

Figure 7: Supply chain graph

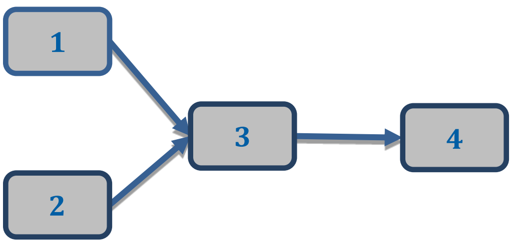
In this case, there are two alternative processes that can be used to make a candle holder, one of them ($P_2$) involving 3D printing.

We distinguish four types of nodes in the supply chain graph, in order to model different flows through the arcs of the graph:

- **Initial nodes**, indexed by the initial set $I \subseteq N$, are those nodes where processes begin.
- **Assembly nodes**, indexed by the assembly set $A \subseteq N$, are those nodes where their ingoing arcs move parts that will be assembled at the node; therefore, it needs all their ingoing arcs active to manufacture some production.
- **Production nodes**, indexed by the production set $D \subseteq N$, are those nodes where any ingoing arc carries a given amount of the same semi-finished product (unlike the assembly nodes); then, production can come from any subset of the ingoing arcs.
- **Market nodes**, indexed by the market set $M \subseteq N$, are those nodes where processes end and production is sold. Unlike the other operations, these nodes do not have production costs and always run in a postponed strategy.

This characterization allows studying multi-product cases through assigning different products to different markets. Notice that the initial, production and assembly nodes can model any supply chain operation except sales (modeled by market nodes). Their differences lie in the relationship with the ingoing arcs.

An instance of the $(OSCS)$ problem should begin by defining a graph containing all the operations we want to evaluate.




| | | |
|---|---|---|
| **1** | Manufacturing piece 1 | *Initial* |
| **2** | Manufacturing piece 2 | *Initial* |
| **3** | Assembly pieces | *Assembly* |
| **4** | Market | *Market* |

**Figure 8: Example of supply chain graph**

Actually, the formulation of the $(OSCS)$ problem allows for complex manufacturing situations, such as several markets $M_i$ or operations $O_i$ with any number of preceding and subsequent operations:

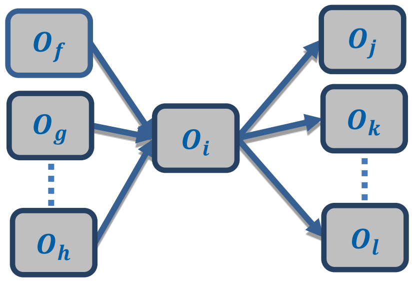



Figure 9: Generic production node

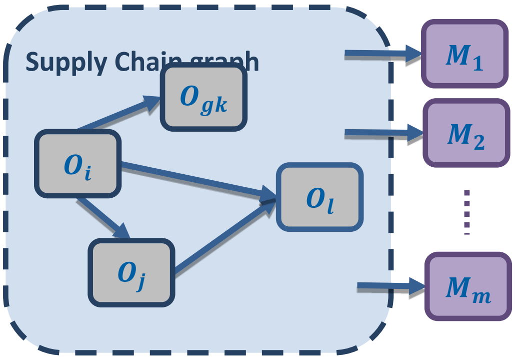



Figure 10: Multi-market graph

## Operations

Every supply chain operation (except the sales to the markets, that is, every node $i \in N \setminus M$) is defined through the following parameters:

- A unitary production cost $c_i$ $(€/unit)$.
- A set-up production cost $f_i$ $(€)$.
- The lifetime of the operation $q_i$ $(unit)$.
- A unitary lead time $l_i$ $(h/unit)$, the time a production unit takes to be processed at operation $i$.

In case there is a significant delay in the transportation of semi-finished goods from node $i$ to node $j$, we can assign a lead time directly to arc $(i,j) \in L$:

- A fixed lead time $t_{ij}$ $(h)$, the time it takes a production flow to arrive at node $j$ from node $i$.

If the supply chain operation $i$ represents a market (i.e., $i \in M$), the associated parameters are:

- A unitary selling price $p_i$ $(€/unit)$.
- A unitary stock-out cost $o_i$ $(€/unit)$.

Aside from the inner characteristics of each node $i \in N$ (selling price and stock-out if $i \in M$, costs and lead times otherwise), some information must be input about the possibility of installing a buffer just before each operation. The buffers installed will be those selected as decoupling points, which is characterized through defining:

- A unitary initial holding cost $h_i$ $(€/unit)$.

- A unitary final holding cost $fh_i$ $(€/unit)$.
- A fixed set-up cost $z_i(€)$.

Initial holding cost $h_i$ will penalize each unit stored at $i$ in speculation, while final holding cost $fh_i$ will penalize each unit stocked at $i$ after demand is done, and the fixed set-up cost $z_i$ computes the cost resulting from installing such a buffer.

Initial and final holding parameters $h_i$ and $fh_i$ do not hold at assembly nodes: as the buffer would be placed before the operation is done (postponed), it is necessary to separate each piece; and the number of pieces assembled for the product must be defined. Denoting each piece by an ingoing arc $(j, i)$, we define:

- A unitary initial holding cost $ah_{ji}$ $(€/unit)$.
- A unitary final holding cost $fah_{ji}$ $(€/unit)$.
- Ratio of pieces/product $r_{ji}(unit)$.

### Flexible and Dedicated Technologies

Especially when studying single product cases, there is a qualitative difference between investing in a dedicated technology or in a flexible one. In both cases, an initial investment must be made for setting up the technology in the supply chain. But the flexible technology will later be able to produce other products, whereas the dedicated one will not.

This difference has been modeled by assuming that flexible technologies automatically amortize by their lifetime $q$, increasing what we call practical production cost $c$ to $\hat{c} = c + f/q$, while its practical fixed cost $f$ is set at $0$. We will denote *practical cost* as the amortized cost input to the model, denoting therefore that these parameters have been modified before being input to the model.

The following figure summarizes the parameters associated with every initial or production operation:

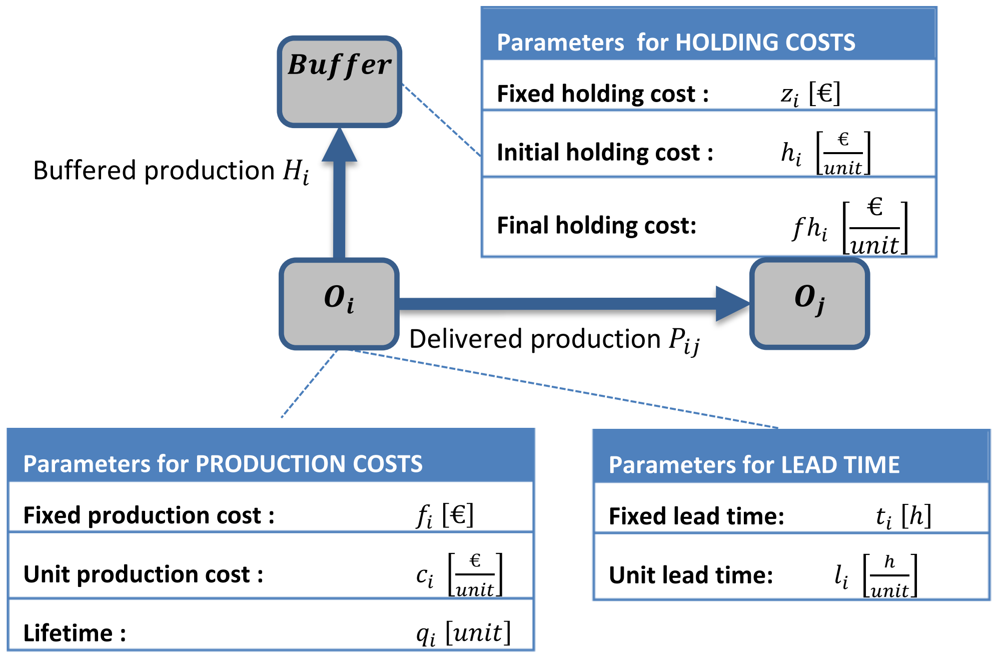



Figure 11: Cost-time parameters

And the next one summarizes the market parameters:

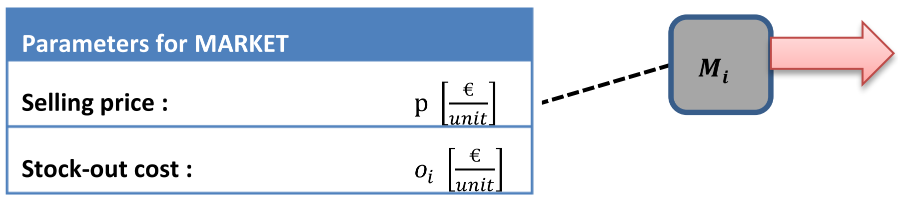



Figure 12: Market parameters

## Stochasticity

Before developing the formulation of the $(OSCS)$ problem, it is necessary to explain our assumptions on the structure of uncertainty in demand and their related parameters. Let $d$ be the random variable associated with the total demand along $n^P$ time periods of equal length $t^P$. As usual in stochastic programming, $d$ is going to be represented in our model by a set of scenarios: $d_s$, $s \in \Omega$, with probability $\omega_s$ and size $s_{set}$. The scenario-generation used in the $(OSCS)$ model relies on the following assumptions:

i. There is a constant large number of potential customers $n^C$ with low purchasing probability $p^C$ at a given time period, which remains constant along all time periods $t = 1 \dots, n^P$.

ii. The demand $d$ is the total number of orders placed by the $n^C$ customers along the total time horizon $T = n^P \cdot t^P$.

iii. The demand $d_s$ of each scenario is evenly distributed over the $n^P$ time periods.

Assumption i. states, under the independency hypothesis, that the probability distribution describing the demand at a given time period $t = 1, \dots, n^P$ is a binomial distribution $B(n^C, p^C)$. As these parameters are constant along every period $t$, we can derive the probability distribution describing the total demand $d$ over the time horizon $T$ as a sum of identically distributed binomials, $d \sim B(n^P \cdot n_c,\ p_c)$.

The Central Limit Theorem allows approximating this binomial with large $n = n^P \cdot n_c$ (the total number of potential of customers) and small probability $p_c$ (the purchasing probability of a single buyer), and it does so by using a normal distribution with parameters $N\big(\mu = n^P \cdot n_c \cdot p,\ \sigma^2 = n^P \cdot n_c \cdot p(1-p)\big)$. Consequently, the approximation

$$d \sim B(n^P \cdot n^C, p^C) \approx N\big(\mu = n^P \cdot n^C \cdot p,\ \sigma^2 = n^P \cdot n^C \cdot p(1-p)\big)$$

can be used to generate the set of scenarios $(d_s, \omega_s)_{s\in\Omega}$ from the normal distribution $N(\mu, \sigma)$ with the estimated (or observed) values of the mean $\mu$ and variance $\sigma^2$ of the total demand $d$.

Once scenarios $(d_s, \omega_s)_{s\in\Omega}$ have been generated, the homogeneous distribution of the demand $d_s$ along every time period $t$ established by assumption iii. allows defining a mean demand rate $\lambda_s = d_s/n^P$ for each time period $t$. To derive the probability distribution, we make the following deduction.

Given a fixed time horizon $T$ and any demand value for some scenario $d_s$, let $n^{P*}$ be a large number of time periods of length $t^{P*} = T/n^{P*}$, such that the probability of giving two or more orders in a given time period is negligible. Then, the Law of Rare Events states that the resulting demand during any of such time periods follows a Poisson distribution $Poiss(\lambda_s^* = d_s/n^{P*})$. Then, if $n^{P*}$ is large enough (i.e., $t^{P*}$ small enough), we can obtain the distribution of the demand at a given time period $t$ of length $t^P$as a sum of the demands over every time period $t^{P*}$ contained in $t^P$, i.e., as the sum of $n^{P*}/n^P$ i.i.d. Poisson random variables of rate $\lambda_s^* = d_s/n^{P*}$; which leads to:

$$\sum_{1}^{n^{P*}/n^P} Poiss\left(\lambda_s^* = \frac{d_s}{n^{P*}}\right) = Poiss\left(\left(\frac{n^{P*}}{n^P}\right) \cdot \left(\frac{d_s}{n^{P*}}\right)\right) = Poiss(\lambda_s = \frac{d_s}{n^P})$$

The purpose of introducing the Poisson distribution $Poiss(\lambda_s)$ for describing the demand at each time period $t^P$ is to incorporate into the model the demand variability's effects on the waiting time at each time period $t^P$. This is done without the need of a multi-period problem with $n^P$ stages, and thus provides all associated savings in terms of model scale and execution time. To this end, the demand at each period $t^P$, $d_s^P \sim Poiss(\lambda_s)$, is discretized through the generation of a set of $Q_s$ realizations $d_{sq}^P$ , $q = 1, \dots, Q_s$ with a known probability $\pi_{sq}$, so that every scenario $s \in S$ will contain a set of realizations $\left(d_{sq}^P, \pi_{sq}\right)_{q \in Q_s}$.

In summary, the stochasticity in the demand is represented in our model by denoting the pairs $(s, q) \in \Omega \times Q_s$, each one with an associated $d_s$, $\omega_s$, $d_{sq}^P$ and $\pi_{sq}$.

| | |
|---|---|
| $\Omega$ : | Scenario set. |
| $Q_s$ : | Realization set. |
| $d_s$: | Demand for scenario $s$ along $T$. |
| $\omega_s$: | Probability for scenario $s$. |
| $Q_s$: | Realization set for scenario $s$. |
| $d_{sq}^P$: | Demand at time period $t^P$ for realization $(s, q)$. |
| $\pi_{sq}$: | Probability of realization $(s, q)$. |

This partition of time in $n^P$ time periods of length $t^P$ aims to somehow model the waiting time of the customers in the supply chain. We will assume that the supplier gathers up some amount of order arrivals $d_{sq}^P$ during a time period $t^P$ and triggers the processes in the supply chain to serve this demand in the next time period, generating a maximum waiting time of $2 \cdot t^P$ :

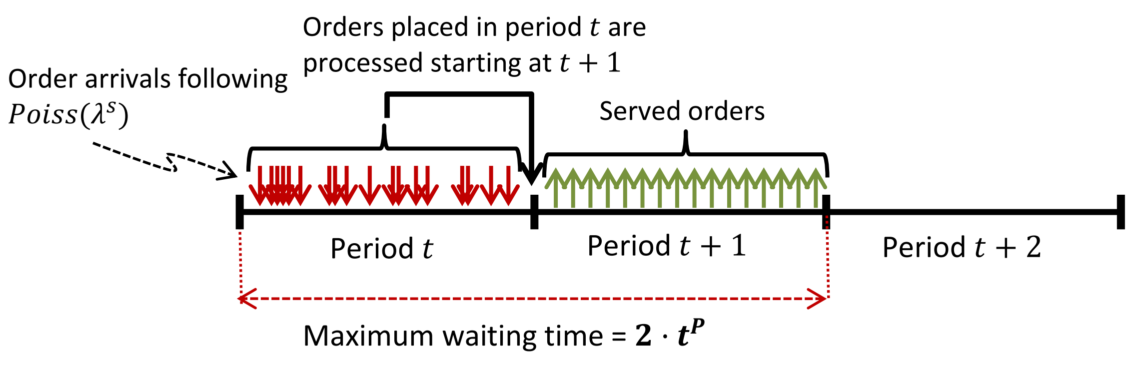



Figure 13: Demand arrivals

# 4. Formulation of the $(OSCS)$ Problem

In the previous section the graph structure, parameters and stochasticity of the $(OSCS)$ problem were presented. In the following sections, the remaining elements of the model (variables, constraints and objective function) are going to be defined and explained. In order to facilitate the understanding of the explanation, and instead of just giving a list with the variables, an understandable description of the conceptual constituents of the model (strategy, production, assembly, market delivery, profit) will be provided while gradually introducing the mathematical elements needed to formulate those conceptual components.

Previous to the development of the model, it is important to establish that the $(OSCS)$ problem is a two-stage stochastic programming problem, where:

- The underlying random variable is the demand at each market.
- The first stage corresponds to all the decisions to be taken before the actual value of the demand is known. These decisions correspond to the manufacturing technology to be deployed, to the customer order decoupling point (CODP) and to the (speculative) production of each operation before the demand is realized.
- The second stage correspond to the recourse action to be taken (postponed production) to fulfill the actual demand.

To our knowledge, the use of the first stage variables / recourse variables for modeling the speculation/postponement levels of production is an original contribution of this work.

## Strategy

Strategy refers to decisions on:

1) Which operations are going to be selected to take part in the deployment of the supply chain.
2) If the selected operations are going to fall under speculation or postponement.

Actually, the first one means identifying the optimal manufacturing technology (whether classical mold or new 3D printing, for instance); and the second refers to the degree of postponement or, in terms of Kemal (2010), the positioning of the customer order decoupling point (CODP) in the supply chain. The unified modeling presented here is for coping with these two issues simultaneously, and it is one of the most relevant contributions of this work.

### Strategic Variables

First, let us define the first stage variable $W^j$ to denote if operation $j$ is selected (active) or discarded from the optimal strategy:

$$W^j = \begin{cases} 0, & \text{if op. j discarded} \\ 1, & \text{if op. j selected} \end{cases}$$

For every (active) operation, variable $Z^j$ determines if that operation corresponds to the CODP of the process that operation belongs to:

$$Z^j = \begin{cases} 0, & \text{if op. j is not a CODP} \\ 1, & \text{if op. j is a CODP} \end{cases}$$

Following the idea introduced in the n-steps models of Lee and Tang (1997) and Ngniatedema et al (2015), we consider that each operation $j$ has its own buffer. Therefore, if operation $j$ is a CODP, we can store the production of node $j$ $H^j$ into its buffer during the speculation phase (1st stage), which will eventually release this production into the postponed phase (2nd stage):

$H^j \in \mathbb{Z}_0^+$ : Production stored at $j \in N \setminus A$ in speculation

The next step is to define the state of the production flow through the link between two consecutives operations $i$ and $j$, where operation $j$ gets the output of operation $i$. To this end, let us define the binary variables $X^{ij}$ and $Y^{ij}$, which denote the activity and strategy (postponement/speculation) of each arc $(i,j)$:

$$X^{ij} = \begin{cases} 0, & \text{if } (i,j) \text{ inactive} \\ 1, & \text{if } (i,j) \text{ active} \end{cases}, \qquad Y^{ij} = \begin{cases} 0, & \text{if } (i,j) \text{ inactive or speculative} \\ 1, & \text{if } (i,j) \text{ postponed} \end{cases}$$

For every active link $X^{ij} = 1$, there will be a given amount of production sent from operation $i$ to operation $j$. Should that flow be speculative ($Y^{ij} = 0$), then the amount of flow is represented by the first-stage variable $P_0^{ij}$. Conversely, if the flow is postponed ($Y^{ij} = 1$), then the flow is represented by the second-stage variable $P_{sq}^{ij}$. Therefore, for each arc $(i,j)$, we define the following variables:

$P_0^{ij} \in \mathbb{Z}_0^+$ : Speculative (deterministic, first-stage variable) flow of arc $(i,j)$

$P_{sq}^{ij} \in \mathbb{Z}_0^+$ : Postponed (stochastic, second-stage variable) flow of arc $(i,j)$ in realization $(s,q)$ of scenario $s$

Throughout the formulation of constraints, we will be interested in pointing to specific subsets of arcs. From now on we denote:

**Destination *j***: subset of arcs with destination node *j* $D(j) := \{(i,k) \in L |\ k = j\}$

**Origin *j***: subset of arcs with origin node *j* $O(j) := \{(i,k) \in L | i = j\}$

In other words, $D(j)$ is the subset of ingoing arcs of $j \in N$, and $O(j)$ is the subset of outgoing arcs. This notation will be extended to sets (e.g., $O(I)$ the subset of outgoing arcs of all Initial operations), and to Cartesian products (e.g., $D(j) \times O(j)$ the subset of pairs of arcs $(i,j) \times (j,k)$ with one common node: destination for the first arc and origin for the second).

We also assume each realization set $Q_s$ of each scenario $s \in \Omega$ to has equal size. We denote $s_{set}$ as the number of scenarios and $q_{set}$ as the number of realizations of each scenario, i.e. $s_{set} = |\Omega|$, $q_{set} = |Q_s|$.

### Strategic Coupling

The following linking constraints establish the relationships between the variables defined so far:

$$P_0^{ij} \leq u \cdot (1 - Y^{ij}) \qquad (i,j) \in L \qquad (1)$$

$$\sum_{(s,q) \in \Omega \times Q_s} P_{sq}^{ij} \leq (u \cdot s_{set} \cdot q_{set}) \cdot Y^{ij} \qquad (i,j) \in L \qquad (2)$$

$$X^{ij} \leq P_0^{ij} + \sum_{(s,q) \in \Omega \times Q_s} P_{sq}^{ij} \leq (u \cdot s_{set} \cdot q_{set}) \cdot X^{ij} \qquad (i,j) \in L \qquad (3)$$

$$H^j \leq u \cdot Z^j \qquad j \in N \setminus A \qquad (4)$$

$$W^j \leq \sum_{k \in O(j)} X^{jk} \leq |O(j)| \cdot W^j \qquad j \in N \setminus M \qquad (5)$$

where $u$ is any upper bound on the total production flow , and $|O(j)|$ is the number of outgoing arcs of node $j$.

Strategic coupling constraints sum a total of $3 \cdot |L| + 2 \cdot |N| - |M| - |A|$ inequality constraints.

### Strategic CODP

The next family of inequalities models the speculation/postponement strategy and where the decoupling points may be placed.

$$(X^{jk} - Y^{jk}) \leq 1 - Y^{ij} \qquad (i,j) \times (j,k) \in D(j) \times O(j), j \in N \qquad (6)$$

$$Z^j \geq Y^{jk} + (X^{ij} - Y^{ij}) - 1 \qquad (i,j) \times (j,k) \in D(j) \times O(j), j \in N \qquad (7)$$

$$Z^j \geq Y^{jk} \qquad (j,k) \in O(I) \qquad (8)$$

$$Z^j \geq (X^{ij} - Y^{ij}) \qquad (i,j) \in D(M) \qquad (9)$$

$$X^{ij} \geq Y^{ij} \qquad (i,j) \in L \qquad (10)$$

Strategic CODP constraints sum a total of $|L| + |O(I)| + |D(M)| + 2 \cdot \kappa$ inequality constraints, where $|O(I)|$ is the number of outgoing arcs of initial nodes, $|D(M)|$ is the number of ingoing arcs of market nodes, and$\kappa$ is the number of destination-origin connected pairs of arcs, i.e.:

$$\kappa = |\{(i,j) \times (k,l) \in L \times L : j = k\}|$$

## Process Flow

Flow refers to those elements of the $(OSCS)$ model that guarantee the coherency of the manufacturing strategy from the point of view of the defined processes. In other words, flow guarantees that the sequence of active operations and links defines a feasible manufacturing process (path) from one initial node to one market. To this end, we need to impose conditions that affect the production flow from the initial to final (market) nodes.

We first need to define the nonnegative integers *Release* and *Final*, holding variables for each operation $j \in N$ and each realization $(s,q) \in \Omega \times Q_s$. These 2nd-stage variables represent the amount of production coming from some precedent operation that is either buffered or released at operation:

- $R_{sq}^j \in \mathbb{Z}_0^+$ : Production released at $j \in N$ in realization $(s,q)$ for scenario $s$
- $F_s^j \in \mathbb{R}_0^+$ : Production finally stored at $j \in N$ in scenario $s$

These variables will be useful when operation $j$ receives some production in a speculative strategy, then operates and distributes them in a postponed strategy. When this situation happens, some production quantity can be held in the buffer of operation $j$.

Unlike the other variables, $F_s^j$ does not need to be integer because it is directly related with the expectation of discrete random variables, which doesn't need to be integer neither (see constraint (13) below).

### Production Flow Equations

Production flow equations model the process flow through nodes. The possibility of putting a decoupling point at a given node $j$ requires generalizing network flow equations, i.e., those that equal all ingoing quantity to all outgoing quantity. Below we show and describe these flow equations for production nodes, and then we will describe analogous equations for initial, assembly and market nodes.

$$\sum_{i \in D(j)} P_0^{ij} = H^j + \sum_{k \in O(j)} P_0^{jk} \qquad j \in D \qquad (11)$$

$$R_{sq}^j + \sum_{i \in D(j)} P_{sq}^{ij} = \sum_{k \in O(j)} P_{sq}^{jk} \qquad j \in D, (s,q) \in \Omega \times Q_s \qquad (12)$$

$$H^j - \sum_{q \in Q_s} \pi_{sq} \cdot R_{sq}^j = F_s^j \qquad j \in D, s \in \Omega \qquad (13)$$

Equation (11) describes the speculative flow equation: all speculative production entering node $j$ must be stored at $H_j$ (if $j$ is the decoupling point) or must be manufactured and delivered speculatively to some outgoing arcs. Equation (13) does this when $j$ is the decoupling point: for any scenario $s \in \Omega$, stored production at $H_j$ is being released by time periods (associated with realizations of demand $d_{sq}^P$ with a known frequency $\pi_{sq}$); and the remaining stock is considered final holding $F_s^j$. Equation (12) describes the postponed flow equation: either the released production (if $j$ is the decoupling point) or the ingoing postponed production (otherwise) must be manufactured and delivered to (some of) all outgoing arcs.

Production flow equations sum a total of $|D| + |D| \cdot |\,\Omega \times Q_s| + |D| \cdot |\Omega|$ equality constraints, where $|D|$ is the number of production nodes.

### Initial Flow Equations

At the initial nodes, it may be useful to know the total production that the node has to supply. We define initial production variables as

$K^j \in \mathbb{Z}_0^+$ : Initial production of node $j \in I$

Initial flow equations are

$$K^j = H^j + \sum_{k \in O(j)} P_0^{jk} \qquad j \in I \qquad (14)$$

$$R_{sq}^j = \sum_{k \in O(j)} P_{sq}^{jk} \qquad j \in I, (s,q) \in \Omega \times Q_s \qquad (15)$$

$$H^j - \sum_{q \in Q_s} \pi_{sq} \cdot R_{sq}^j = F_s^j \qquad j \in I, s \in \Omega \qquad (16)$$

Initial flow equations are a simplification of production ones, substituting ingoing arcs (that do not exist) with an initial production variable $K^j$.

Initial flow equations sum a total of $|I| + |I| \cdot |\, \Omega \times Q_s| + |I| \cdot |\Omega|$ equality constraints, where $|I|$ is the number of initial nodes.

### Assembly Flow Equations

For the assembly nodes, we need to distinguish the holding and released production by its ingoing arcs, i.e., by the different pieces that are assembled at the node. Then, we also define special variables for the assembly flow equations:

$AH^{ij} \in \mathbb{Z}_0^+$ : Production of piece $i$ stored at $j \in A$ in speculation.

$AR_{sq}^{ij} \in \mathbb{Z}_0^+$ : Production of piece $i$ released at $j \in A$ in realization $(s, q)$.

$AF_s^{ij} \in \mathbb{R}_0^+$ : Production of piece $i$ finally stored at $j \in A$ in scenario $s$.

The assembly flow equations are the following:

$$P_0^{ij} = AH^{ij} + r_{ij} \cdot \sum_{k \in O(j)} P_0^{jk} \qquad (i,j) \in D(A) \qquad (17)$$

$$AR_{sq}^{ij} + P_{sq}^{ij} = r_{ij} \cdot \sum_{k \in O(j)} P_{sq}^{jk} \qquad (i,j) \in D(A), (s,q) \in \Omega \times Q_s \qquad (18)$$

$$AH^{ij} - \sum_{q \in Q_s} \pi_{sq} \cdot AR_{sq}^{ij} = AF_s^{ij} \qquad (i,j) \in D(A), s \in \Omega \qquad (19)$$

$$AH^{ij} \leq u \cdot Z^j \qquad j \in A \qquad (20)$$

Where $r_{ij}$ is the conversion factor between pieces $(i, j)$ and units $(j, k)$.

Assembly flow equations sum a total of $|D(A)| + |D(A)| \cdot |\, \Omega \times Q_s| + |D(A)| \cdot |\Omega|$ equality constraints and $|A|$ inequality ones, where $|D(A)|$ is the number of ingoing arcs of assembly nodes.

### Market Flow Equations

Production finally arrives at markets, where it is sold to fulfill the ${d^P}_{sq}^j$ orders of the realization $q \in Q_s$ of each scenario $s \in \Omega$. Depending on the relationship between production and demand, both an excess production and stock-out situation may happen. We define nonnegative integer variables sales and stock-out as follows:

$S_{sq}^{j} \in \mathbb{Z}_0^+$ Sales in market $j \in M$ and realization $(s,q)$

$O_{sq}^{j} \in \mathbb{Z}_0^+$ Stock-out in market $j \in M$ and realization $(s,q)$

The market flow equations are:

$$\sum_{i \in D(j)} P_0^{ij} = H^j \qquad j \in M \qquad (21)$$

$$R_{sq}^{j} + \sum_{i \in D(j)} P_{sq}^{ij} = S_{sq}^{j} \qquad j \in M, (s,q) \in \Omega \times Q_s \qquad (22)$$

$$d^{P}{}_{sq}^{j} = O_{sq}^{j} + S_{sq}^{j} \qquad j \in M, (s,q) \in \Omega \times Q_s \qquad (23)$$

$$H^j - \sum_{q \in Q_s} \pi_{sq} \cdot R_{sq}^{j} = F_s^j \qquad j \in M, s \in \Omega \qquad (24)$$

Market flow equations sum a total of $|M| + 2 \cdot |M| \cdot |\,\Omega \times Q_s| + |M| \cdot |\Omega|$ equality constraints, where $|M|$ is the number of market nodes.

### Time Equations

Time equations model the lead time the postponed production takes to arrive at markets, i.e., both operations and delivery lead time of the paths from decoupling point to markets. We define nonnegative continuous second-stage variables:

$T_{sq}^{j} \geq 0$ : Postponement lead time until $j \in N$ in realization $(s,q)$.

$U_{sq}^{j} \geq 0$ : Idle time in market $j \in M$ in realization $(s,q)$.

$V_{sq}^{j} \geq 0$ : Saturation time in market $j \in M$ in realization $(s,q)$.

$$T_{sq}^{j} \geq l_j \cdot P_{sq}^{ij} + t_{ij} \cdot Y^{ij} + T_{sq}^{i} \qquad (i,j) \in L, (s,q) \in \Omega \times Q_s \qquad (25)$$

$$T_{sq}^{j} = l_j \cdot \sum_{k \in O(j)} P_{sq}^{jk} \qquad j \in I, (s,q) \in \Omega \times Q_s \qquad (26)$$

$$t^P - T_{sq}^{j} = U_{sq}^{j} - V_{sq}^{j} \qquad j \in M, (s,q) \in \Omega \times Q_s \qquad (27)$$

$$\sum_{q \in Q_s} \pi_{sq} \cdot (U_{sq}^j - V_{sq}^j) \geq 0 \qquad j \in M, s \in \Omega \qquad (28)$$

$$\sum_{(s,q) \in \Omega \times Q_s} \omega_s \cdot \pi_{sq} \cdot V_{sq}^j \leq \alpha \cdot t^P \qquad j \in M \qquad (29)$$

Equation (25) describes the current lead time at $j$ by taking previous lead times and adding both the delivery and manufacturing times of operation $j$. Equation (26) is the initialization of this recursive computation.

Here we could have bounded $T_{sq}^j$ by the period length $t^P$ at the market nodes, but as we are considering several demand realizations $(s, q)$ for the same scenario $s$, it is natural to allow the system to take some extra time to deliver postponed production whenever this time can be recovered in other realizations of same scenario.

In this way, we take saturation time $V_{sq}^j$ and idle time $U_{sq}^j$ for each market $j$ and realization $(s, q)$ from equation (27). We make two upper bounds at equations (28) and (29): at (28) we state that some extra time can be taken as long as it can be recovered at other realizations from the same scenario $s$ and market $j$; while in (29) we bound the expected saturation time by a fraction $\alpha$ of the period length $t^P$. This last parameter $\alpha$, which we will call *saturation rate*, will model the flexibility of delivering "a little bit later" when needed.

Time equations sum a total of $|M| \cdot |\,\Omega \times Q_s| + |I| \cdot |\,\Omega \times Q_s|$ equality constraints and $|L| \cdot \,|\,\Omega \times Q_s| + |M| \cdot |\Omega| + |M|$ inequality ones.

## Objective Function

All constraints declared above define the feasible set of solutions of $(OSCS)$. From among them all, we are interested in finding those that maximize the total expected profit of running the supply chain. This expected total profit is composed of two terms:

| $\boldsymbol{TotalProfit}$: | | |
|---|---|---|
| 1st stage | Operation set-up costs | $-\sum_{j \in N \setminus M} f_j \cdot W^j$ |
| | Decoupling Point set-up costs | $-\sum_{j \in N} z_j \cdot Z^j$ |
| | Speculative production costs | $-n^P \cdot \sum_{(i,j) \in L} c_j \cdot P_0^{ij}$ |

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| | | |
|---|---|---|
| | Initial holding costs | $-n^P \cdot \sum_{j \in N \setminus A} h_j \cdot H^j$ |
| $2^{st}$ stage | Expected $2^{nd}$ stage profit | $\mathbb{E}_s\left(Profit(s\,; X, Y, Z, W, K, P_0, H, AH)\right)$ |

The $1^{st}$ stage terms are composed of investment costs, for operations and decoupling points as well as for speculative production costs and initial holding costs. Given a scenario $s \in \Omega$ and $1^{st}$ stage decisions $(X\,, Y,\ Z,\ W,\ K,\ P_0,\ H,\ AH)$, the $2^{nd}$ stage term is also composed of two main terms:

| $\boldsymbol{Profit(s\,; X, Y, Z, W, P_0, H, AH)}$: | | |
|---|---|---|
| $2^{nd}$ stage costs of scenario $\boldsymbol{s}$ | Final holding costs | $-n^P \cdot \sum_{j \in N \setminus A} fh_j \cdot F_s^j$ |
| | Final assembly holding costs | $-n^P \cdot \sum_{(i,j) \in D(A)} fah_{ij} \cdot AF_s^{ij}$ |
| $2^{nd}$ stage profit of scenario $\boldsymbol{s}$ and realization $\boldsymbol{q}$ | Postponed production costs | $\mathbb{E}_q\left(-n^P \cdot \sum_{(i,j) \in L} c_j \cdot P_{sq}^{ij}\right)$ |
| | Stock-out costs | $\mathbb{E}_q\left(-n^P \cdot \sum_{j \in M} o \cdot O_{sq}^j\right)$ |
| | Sales benefits | $\mathbb{E}\left(n^P \cdot \sum_{j \in M} p \cdot S_{sq}^j\right)$ |

This formula is composed of the final holding costs and the expectation of postponed production costs, stock out costs and sales benefits, over the realizations $q \in Q_s$ of scenario $s$.

All the expected terms have been computed as the weighted sum of its quantities. Finally, the aggregation of all terms is computed as an objective function:

$$\max TotalProfit = n^P \cdot \sum_{j \in M} \sum_{(s,q) \in \Omega \times Q_s} \omega_s \cdot \pi_{sq} \cdot p \cdot S_{sq}^j \qquad \text{Sales benefits}$$

$$-n^P \cdot \sum_{j \in M} \sum_{(s,q) \in \Omega \times Q_s} \omega_s \cdot \pi_{sq} \cdot o \cdot O_{sq}^j \qquad \text{Stock-out costs}$$

$$-n^P \cdot \sum_{j \in N \setminus A} \sum_{s \in \Omega} \omega_s \cdot fh_j \cdot F_s^j \qquad \text{Final holding costs}$$

$$-n^P \cdot \sum_{\substack{(i,j)\in L \\ j\in A}} \sum_{s\in\Omega} \omega_s \cdot fah_{ij} \cdot AF_s^{ij}$$ Final assembly holding costs

$$-n^P \cdot \sum_{j\in N\setminus A} h_j \cdot H^j$$ Initial holding costs

$$-n^P \cdot \sum_{\substack{(i,j)\in L \\ j\in A}} ah_{ij} \cdot AH_s^{ij}$$ Initial assembly holding costs

$$-n^P \cdot \sum_{(i,j)\in L} \sum_{(s,q)\in\Omega\times Q_s} \omega_s \cdot \pi_{sq} \cdot c_j \cdot P_{sq}^{ij}$$ Postponed production costs

$$-n^P \cdot \sum_{(i,j)\in L} c_j \cdot P_0^{ij}$$ Speculative production costs

$$-\sum_{j\in N} z_j \cdot Z^j$$ CODP set-up costs

$$-\sum_{j\in N\setminus M} f_j \cdot W^j$$ Operation set-up costs

## Final Model $(OSCS)$

The model $(OSCS)$ can be expressed as:

$$(OSCS)\begin{cases} \max & TotalProfit \\ s.t.: & \\ Str.Coupling & (1)-(5) \\ Str.CODP & (6)-(10) \\ Pro.Flow & (11)-(13) \\ Ini.Flow & (14)-(16) \\ Ass.Flow & (17)-(20) \\ Mar.Flow & (21)-(25) \\ Time & (26)-(29) \\ Variable\ Domains & \end{cases}$$

Problem $(OSCS)$ is a mixed integer linear two-stage stochastic program with the following number of constraints:

**Equality:** $(|D|+|I|+|D(A)|+|M|)\cdot(1+s_{set})+(|D|+2\cdot|I|+|D(A)|+3\cdot|M|)\cdot s_{set}\cdot q_{set}$

**Inequality:** $4\cdot|L|+2\cdot|N|+|O(I)|+|D(M)|+2\cdot|L|\cdot s_{set}\cdot q_{set}+2\cdot\kappa$

being $s_{set} = |\Omega|$, $q_{set} = |Q_s|$, and $\kappa$ the number of pair of arcs connected, as defined before, i.e.:

$$\kappa = |\{(i,j) \times (k,l) \in L \times L : j = k\}|$$

We summarize here a table with all variables of $(OSCS)$:

| Variable | Symbol | Stage | Domain | Size |
|---|---|---|---|---|
| **Selected Arc** | $X^{ij}$ | $1^{st}$ | $\{0,1\}$ | $\|L\|$ |
| **Speculation/Postponement Strategy** | $Y^{ij}$ | $1^{st}$ | $\{0,1\}$ | $\|L\|$ |
| **Decoupling Point** | $Z^{j}$ | $1^{st}$ | $\{0,1\}$ | $\|N\|$ |
| **Selected Operation** | $W^{j}$ | $1^{st}$ | $\{0,1\}$ | $\|N \setminus M\|$ |
| **Speculative Production** | $P_0^{ij}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|L\|$ |
| **Initial Production** | $K^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|I\|$ |
| **Initial Holding** | $H^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|N \setminus A\|$ |
| **Initial Assembly Holding** | $AH^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|D(A)\|$ |
| **Postponed Production** | $P_{sq}^{ij}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|L \times \Omega \times Q_s\|$ |
| **Released Production** | $R_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|(N \setminus A) \times \Omega \times Q_s\|$ |
| **Released Assembly Production** | $AR_{sq}^{ij}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|D(A) \times \Omega \times Q_s\|$ |
| **Sales** | $S_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Stock-Out** | $O_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Postponement Lead Time** | $T_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|N \times \Omega \times Q_s\|$ |
| **Idle Time** | $U_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Saturation Time** | $V_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Final Holding** | $F_s^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|(N \setminus A) \times \Omega\|$ |
| **Final Assembly Holding** | $AF_s^{ij}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|D(A) \times \Omega\|$ |

**Table 10: Table of variables**

The $(OSCS)$ model contains a total of:

- $2 \cdot |L| + 2 \cdot |N| - |M|$ **binary 1$^{st}$ stage** variables.
- $|L| + |I| + |N| - |A| + |D(A)|$ **nonnegative integer 1$^{st}$ stage** variables.

- $(|L| + |N| - |A| + |D(A)| + 2 \cdot |M|) \cdot |\Omega \times Q_s|$ **nonnegative integer 2nd stage** variables.
- $(|N| - |A| + |D(A)|) \cdot |\Omega| + (|N| + 2 \cdot |M|) \cdot |\Omega \times Q_s|$ **nonnegative continuous 2nd stage** variables.

# 5. Computational Implementation

The $(OSCS)$ model has been coded in AMPL[1] (A Modeling Language for Mathematical Programming), a high-level language for mathematical optimization problems. The reader who is familiar with this language can skip reading the *Introduction* section and proceed to the sections *Demand generation* and *Main file*.

## Introduction

Given an optimization problem and a data instance for this problem, AMPL calls a solver that finds the optimal solution by reading the intermediate file produced by AMPL and applies the appropriate algorithm. We will use the solver CPLEX, an optimization software package specialized in mixed integer linear problems (MILP), such as the $(OSCS)$ model.

There are three kinds of files that can be used with AMPL:

- **.mod** files, where the optimization model is defined, including the declaration of sets, parameters, variables, objective function and constraints.
- **.dat** files, where the values of the data (sets and parameters) declared in the model (.mod file) are implemented.
- **.run** files, where problem is executed. In these files, we can also compute commands to manage the inputs and outputs of the model.

Each instance of the $(OSCS)$ problem is defined through a specific .dat file that contains all the parameters defining each operation and the information about demand scenarios.

### Model : file OSCS.mod

This file contains the formulation of $(OSCS)$ explained in Chapter 4. It is split into five categories:

- **Set declaration**. Here we first declare the sizes of sets, which are generated.

[1] A free version of AMPL IDE with CPLEX can be downloaded at http://ampl.com/try-ampl/download-a-free-demo/.

- **Parameter declaration**. Then we declare all parameters we are going to input to the model.
- **Variable declaration**. Now variables are created with a given domain. For computational efficiency, all 2nd stage variables have been linearly relaxed. This relaxation is supported while those variables do not correspond to any strategic decision but are only realizations of what has been decided in the 1st stage.
- Objective function.
- **Constraints**. Finally, the set of constraints are defined.

### Dataset : file instance.dat

This file inputs a given graph configuration with a cost-time dataset. Notice that (1) the distribution node set is not declared but generated by all nodes not contained in the initial, assembly or market sets; and (2) some parameter families with the same domain, i.e., $c_i$ and $f_i$, may be jointly declared.

## Demand Generation

Aside from characterizing a supply chain, a dataset containing demand information, time intervals and service level must be input. We differentiate here two levels of demand information: demand scenarios and realizations for each scenario.

- Demand scenarios consist of a set of demand values with a given probability of the total demand along the time horizon. This information can be delivered via a historical dataset of sales, via forecast or by directly assuming a probability distribution with given parameters. In both theoretical and real cases, we have assumed a normal distribution with some mean $\mu$ and standard deviation $\sigma$ before then proceeding to generate a random sample for this distribution.
- Given a scenario with an associated demand value, we need to extract information about their realizations, i.e., which values are more likely to happen in each time period and how probable they are. In Chapter 3, we argued that, given a number of time periods $n^P$ and the demand value of a scenario $d_s$, the demand realizations of that scenario follow a Poisson distribution of rate $\lambda_s = d_s/n^P$ in accordance with the Law of Rare Events.

We therefore proceeded to code two .run files which generate that information about demand: the first one generates and sorts demand scenarios, and the second one, given a number of periods $n^P$ and these demand values, computes realization sets for each scenario.

### Generator gen_demand_initial.run

This file generates the $s_{set}$ values of $d_s, s = 1 \ldots s_{set}$. The simplest way to do that is by putting a random seed and generating $s_{set}$ values of the distribution, with the same probability $\omega_s = 1/s_{set}$. In order to obtain a set of integer non-negative scenarios, we used the expression $\max\{0, floor(Normal(\mu, \sigma))\}$ to generate the values of $d_s$. For $s_{set}$ to be large enough, the distribution of $d_s$ obtained this way is a fair approximation of the original normal distribution.

Finally, in this file there is a procedure to sort these values in ascending order for convenience. A bubble sort algorithm has been coded to do this instead of a quick sort or merge sort, which are more efficient but harder to code and implement in a language such as AMPL.

Script *gen_demand_initial.run* is called once at the beginning of every run, and we also use this script to declare some auxiliary parameters to help the second demand generator script described below.

### Generator gen_demand_loop.run

Unlike the previous generator, this one is designed to be executed several times inside a loop, and it will return different outputs for different numbers of time periods $n^P$. This code is basically a loop over the scenario set, which does two steps for each scenario $s$:

- Compute a discretization of $Poiss(\lambda_s = d_s/n^P)$ into a finite set $W_s$ of realizations of size $q_{set}$.
- Compute the probabilities $\pi_{sq}$ of the resulting set and weigh them to sum 1.

The first step generates $W_s$ by discretizing an approximation of the 95% confidence interval of $Poiss(\lambda_s)$, which is given by taking the 95% confidence interval of a normal distribution $[\mu - 1.96 \cdot \sigma, \mu + 1.96 \cdot \sigma]$ with parameters $\mu = \lambda_s, \sigma = \sqrt{\lambda_s}$.

```
If  λs + 2√λs ≤ q_set  then
  Let Ws upper bound := q_set
     Else Let Ws upper bound  u^Ws := λs + 2√λs
End If
If  λs − 2√λs ≤ 0  then
  Let Ws lower bound := 0
     Else Let Ws lower bound  l^Ws := λs − 2√λs
End If
```

Compute a uniform discretization of $[l^{W_s},\ u^{W_s}]$ of size $q_{set}$

The second step computes the probability of each realization of $W_s$, assuming they follow a $Poiss(\lambda_s)$ distribution. Then, since $W_s$ is a finite subset of the all possible realizations, these probabilities must be weighed to sum 1. In a first approach, we tried to compute recursively all realizations for each scenario by computing:

```
Let  PI_0 = Prob(X = 0) = e^{-λ_s}
For  i = 1 … u^{W_s}  do
      Let  PI_i = Prob(X = i) = PI_{i-1} · (λ_s / i)
End For
```

Although this method is theoretically correct, it is affected by numerical errors caused by (a) large values of $\lambda_s$ such that $Prob(X = 0) \simeq 0$, which sets $Prob(X = i)$ to 0 for all $i$; and also by (b) small values of $\lambda_s$ that cause "Infinity". Instead of this, we have implemented the following numerical method to avoid these errors:

```
If  λ_{s0} = 0  then
      Let  PI_0 = e^{-λ_s}
   Else Let PI_0 = 1
End If
For  i = 1 … λ_{s0}  do
      Let  PI_0 = PI_0 · e^{-λ_s / λ_{s0}} · (λ_s / i)
End For
For  q = 1 … q_{set}  do
      For  i = (λ_{s q-1} + 1) … λ_{sq}  do
            Let  PI_q = PI_q · e^{-λ_s / λ_{sq}} · (λ_s / i)
   End For
End For
```

```
For q = 0 … q_set do
    Let π_sq = PI_q / Σ_{k=1}^{q_set} PI_k
End For
```

This method works much better than the previous one, but also it causes numerical errors for large values of $\lambda_s$ (about $\lambda_s \geq 4000$). In these cases (in fact, for $\lambda_s \geq 100$), we assume $\lambda_s$ is big enough to approximate the Poisson distribution by a normal distribution with parameters $\mu = \lambda_s, \sigma = \sqrt{\lambda_s}$ by the Central Limit Theorem, instead of computing the Poisson probabilities.

## Main File

Finally, a file *"main.run"* is a script that solves the sequence of a given problem with different values of $n^P, t^P, \alpha$ by generating specific demand data and solving the model inside a loop. It contains the following steps:

- Header: model and cost-time datasets are loaded here, and solver options are configured.
- 1st stochastic generation: here stochastic sizes $s_{set}, q_{set}$ and demand parameters $\mu, \sigma$ are fixed; then the first demand generator *"gen_demand_initial.run"* is called.
- Loop generation: values of $n^P, t^P, \alpha$ that need to be tested are declared here.
- Loop execution: at each iteration, the second demand generator *"gen_demand_loop.run"* is called, so demand realizations $\lambda_{sq}$ and their probability $\pi_{sq}$ are generated. Then, the problem is solved and the *"printf_results.run"* script is called to return the output to an external file.

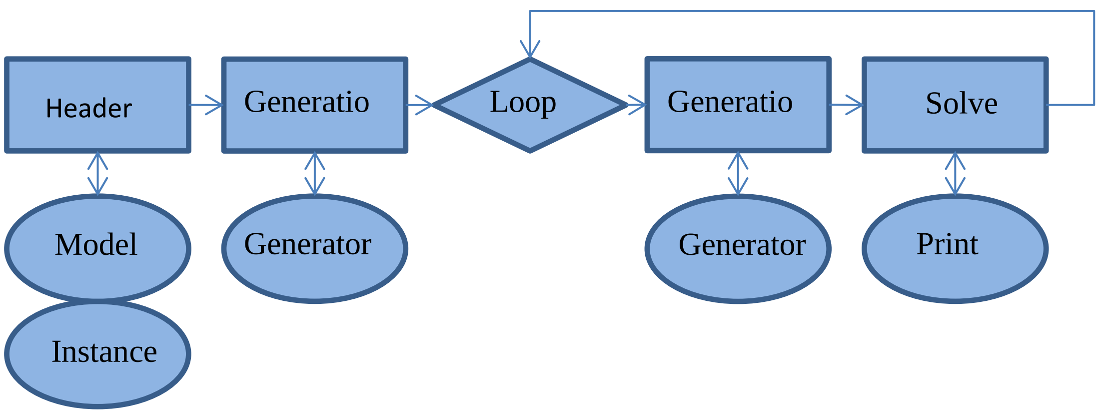


Figure 14: Main file diagram

### Output printf_results.run

This file is basically an aggregate of print commands that outputs the desired results of the statistics to an external file for all instances solved in the loop. It can be extremely useful for writing data in a given way in order to input it later in other software. It also detects some specific behavior of the optimal by slightly changing some parameters.

# 6. Case Studies

In this section, we develop case studies aimed at introducing 3D printing technologies and an optimal degree of postponement for supply chains in automotive, craft retail and toy industries. They are all supported by the data of real companies. The first two test cases were proposed by Accenture and the third one by FCIM.

## Postponement Application in Automotive Spare Parts Industry

A European automotive manufacturing company supplies spare parts to a given number of car repair shops located at different places. New technological developments have shortened the life cycles of an increasing number of products in the automotive spare parts manufacturing sector, which intrinsically has a highly variable demand.

Currently, the company purchases some quantity of each part speculatively and delivers it to each shop before demand is known. Customers need to be catered to in a given time and stock-out situations suppose long-term costs, so large stocks of parts are often generated at shops.

The aim of this study, proposed by Accenture, is to assess the introduction of 3D printing machines into car repair shops versus the current outsourcing strategy. We do so by considering a speculation/postponement strategy decision and some service levels, such as catering before maximum time $(2 \cdot t^P)$ without allowing saturation (low $\alpha$ level). We show a supply chain graph that contains

**Market**

$M_l^p$ One market node for each location $l$ and product $p$

**Current**

$P_p$ One purchasing node (initial) from the supplier for each product $p$

**Alternative**

$3D_l$ One 3D printing node (initial) for each location $l$

The associated graph, aggregated by locations $l$ and products $p$ is:

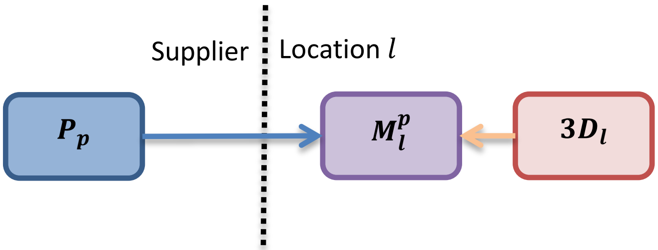


Figure 15: Supply chain graph of automotive case

This simple model can be used to compare the production costs of 3D printing techniques against the holding and stock-out costs of the current supply chain strategy based on injection technology, for any spare part able to be made by 3D printing technology.

This model is made under the following assumptions:

1. Markets $M_l$ (purple) are composed of a set of car repair shops located at places indexed by $l$.
    a. We assume each market to have an expected demand $\mu_l$ with a high deviation $\sigma_l = 0.5 \cdot \mu_l$.
    b. The benefit of delivering one unit (selling price) is $p = 50€$.
    c. The stock-out unit cost is $o = 20€$.
2. Current purchasing process (blue) is composed of a Purchasing node $P_p$ for each location $p$.
    a. As this operation is subcontracted, we aggregate both production and delivery time into a unit cost $c = 4.74\ €/unit$.
    b. The delivery time from purchasing to the market is modeled at arcs $P_p \rightarrow M_l^p$ by a fixed time parameter $t = 7\ days = 168\ h$ for every location $l$ and product $p$.
    c. All holding costs (decoupling point set-up $z$, unit initial holding $h$, unit final holding $fh$) are zero, because it is a subcontracted operation.
3. Alternative process (red) is composed of a specialized 3D printing node $3D_l$ for each location $l$.
    a. Each node consist of a number $N$ of 3D printing machines running in parallel in the car repair shop $l$, with set-up cost $f$ for each machine and production cost $c_p$ for each product $p$.
    b. As the purchasing operation, this operation has no holding costs.

As 3D printing technologies are flexible (they can run for any product $p$), we assume fixed costs $f$ of nodes $3D_l$ are automatically amortized by their lifetime $q$, increasing their practical production cost from $c$ to $\hat{c} = c + f/q$ and reducing their practical fixed cost $\hat{f}$ to $0$.

Then, the number of 3D printing machines working in parallel, say $N$, does not affect the practical production cost $\hat{c} = c + (N \cdot f)/(N \cdot q)$, but it affects the practical production lead time $\hat{l} = l/N$. The decision of how many 3D printers to install must be made before running the model by putting an appropriate value of $\hat{l}$. This creates a trade-off between the initial investment in this technology $f \cdot N$ and its practical production lead time $\hat{l} = l/N$.

Before computing and solving this model, we can exploit its structure by decomposing it into products in order to generate a reduced instance:

- All operations are differentiated by products $p$ – except for the 3D printing ones, which have no set-up production costs or set-up holding costs. If we make the extra assumption "Each 3D printing machine installed will manufacture a single product", the resulting problem is thus separable by products.
- This is due to the time constraints of postponement: a single machine may take too long to manufacture given quantities of several products, but instead it can manufacture each of them over a time period. This extra assumption guarantees that the resulting number of 3D printing machines in the optimal of separated problems will be greater than or equal to those in the original problem.
- Also, each location $l$ must be served independently of the others. As the purchasing operations have no production or holding set-up costs, we can face each location as a different problem.

We have solved an instance for one location and one product with the following dataset:

1. Production costs dataset:

| Operations | Node | Production costs $c_i$ $[€/unit]$ | Set-up costs $f_i$ $[€]$ | Life cycle $\bar{Q}_i$ $[unit]$ |
|---|---|---|---|---|
| **Purchasing $P$** | 1 | 4.74 | 0 | – |
| **3D Printing $3D$** | 2 | 6 | 500000 | 1500000 |
| **Market $M$** | 3 | – | – | – |

Table 11: Production costs of automotive case

2. Holding costs dataset:

| Operations | Node | Holding costs $h_i$ $[€/unit]$ | Final holding costs $fh_i$ $[€/unit]$ | Set-up costs $z_i$ $[€]$ |
|---|---|---|---|---|
| **Purchasing $P$** | 1 | 0 | 0 | 0 |
| **3D Printing $3D$** | 2 | 0 | 0 | 0 |

| **Market $M$** | 3 | 0.1 | 1.5 | 2000 |
|---|---|---|---|---|

Table 12: Holding costs of automotive case

3. Variable lead time dataset:

| Node | Variable lead time $l_i$ $[h/unit]$ |
|---|---|
| **1** | 0 |
| **2** | 12 |
| **3** | – |

Table 13: Variable lead times of automotive case

4. Fixed lead time dataset:

| Arc | Fixed lead time $t_i$ $[h]$ |
|---|---|
| $(\mathbf{1},\mathbf{3})$ | 168 |
| $(\mathbf{2},\mathbf{3})$ | 0 |

Table 14: Fixes lead time of automotive case

5. Market dataset:

| Node | Selling price $p_i$ $[€/unit]$ | Stock-out costs $o_i$ $[€/unit]$ |
|---|---|---|
| **3** | 50 | 20 |

Table 15: Market dataset of automotive case

We have generated a sample of 10 scenarios with 5 period realizations each, following a normal distribution with parameters $N(\mu = 7000, \sigma = 3500)$. We fix $\alpha = 0.05$ and study two maximum delivery cases: 4 hours ($t^P = 2\,h$), and 12 hours ($t^P = 6$), over a horizon of one year. For each case, we make a critical analysis for the number $N$ of 3D machines installed.

This instance has 9 binary variables (1st stage), 1407 nonnegative integer variables (7 of 1st stage, 1400 of 2nd stage), 1060 nonnegative continuous variables (2nd stage), 1,463 equality constraints and 818 inequality constraints. The next cases have been resolved while allowing a relative MIP gap of 0.01.

## Case 1: Maximum Delivery Time of 4 Hours

If we want to serve customers within a maximum delivery time of 4 hours, the length of time periods must be set to $t^P = 2\,h$. Also, we do not allow the chain to be saturated if postponed, so we must fix a low saturation rate such as $\alpha = 5\%$, i.e., we allow the system to take a maximum extra time $0.05 \cdot t^P$, if needed. The sampled expected rate of order arrivals for a period of 2 hours is $\mathbb{E}(\lambda_s) = 1.54\,u$.

The following tables show how the optimal process selection and speculation/postponement strategy changes in terms of the number of 3D printers installed $N$, as well as the total profit, the stock level per period at market and the expected values of stock-out and final holding per period. Operations selected as decoupling points are plotted with a dashed outline.

| 3DP machines | Strategy | Profit | Stock | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $N \leq 3$ | $P_p$ → $M_l^p$ | 260,861€ | $3\ u/p$ | $0\ u/p$ | $1.46\ u/p$ |
| $10 \leq N \leq 14$ | $P_p$ → $M_l^p$ ← $3D_l$ | 282,651€ | $1\ u/p$ | $0\ u/p$ | $0.46\ u/p$ |
| $N \geq 16$ | $M_l^p$ ← $3D_l$ | 293,991€ | $0\ u/p$ | $0\ u/p$ | – |

Table 16: Optimals of automotive case 1

The solutions show how the strategy and processes change, depending on the number of 3D printing machines installed. As time limitations allow it, it is preferable to use a postponed strategy and supply the remaining demand by purchasing some quantity through speculation.

An amount of 3 or fewer 3D printing machines are unable to supply production, so the optimal is to put the decoupling point at market and then purchase speculatively in order to maintain a stock level of 3 units each, $t^P = 2$ hours. The expected total profit along the time horizon will be 260,861€, with an expected stock-out per period of 0 units and an expected final holding of 1.46 units per period.

The optimal solution with 10 to 14 machines is to use both processes. The 3D printing operation is a decoupling point that manufactures an expected postponed production of 0.54 units per period. The remaining demand is served speculatively through purchasing in order to maintain a stock-level of 1 unit per period at market, which is also decoupling point. The expected total profit along the time horizon will be 282,651€, with an expected stock-out per period of 0 units and an expected final holding of 0.46 units per period.

16 or more machines can manufacture all production in postponement during delivery time. Moreover, it is optimal. The 3D printing operation is the decoupling point, and it delivers an expected postponed production of 1.54 units per period. The expected total profit along the time horizon will be 293,991€, with an expected stock-out per period of 0 units and an expected final holding of 0 units per period.

### Case 2: Maximum Delivery Time of 12 Hours

If we want to serve customers with a maximum delivery time of 12 hours, the length of time periods must be set to $t^P = 6\ h$. Also, we do not allow the chain to be saturated if it is postponed, so we must fix a low saturation rate such as $\alpha = 5\%$, i.e., we allow the system to take a maximum extra time of $0.05 \cdot t^P$, if needed. The sampled expected rate of arrivals per period of 6 hours is $\mathbb{E}(\lambda_s) = 4.61\ u$. Operations selected as decoupling points are plotted with a dashed outline.

| 3DP machines | Strategy | Profit | Stock | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $N \leq 2$ | $P_p$ → $M_l^p$ | 260,365€ | $8\ u/p$ | $0.06\ u/p$ | $3.5\ u/p$ |
| $8 \leq N \leq 12$ | $P_p$ → $M_l^p$ ← $3D_l$ | 286,676€ | $4\ u/p$ | $0\ u/p$ | $0.93\ u/p$ |
| $N \geq 14$ | $M_l^p$ ← $3D_l$ | 290,802€ | $0\ u/p$ | $0\ u/p$ | – |

Table 17: Optimal solutions of automotive case 2

As before, the strategy and processes change in terms of the number of 3D printing machines installed.

2 or fewer machines are unable to supply production, so the optimal is to put the decoupling point at market and purchase speculatively in order to maintain a stock level of 8 units each $t^P = 6$ hours. The expected total profit along the time horizon will be 260,365€, with an expected stock-out per period of 0.06 units and an expected final holding of 3.5 units per period.

The optimal solution with 8 to 12 machines is to use both processes: the 3D printing operation is a decoupling point that manufactures an expected postponed production of 0.61 units per period. The remaining demand is purchased speculatively to maintain a stock-level of 4 units per period at market, which is also a decoupling point. The expected total profit along the time horizon will be 286,676€, with an expected stock-out per period of 0 units and an expected final holding of 0.93 units per.

14 or more machines can manufacture all production in postponement during delivery time. Moreover, it is optimal. The 3D printing operation is a decoupling point, and it delivers an expected postponed production of 4.61 units per period. The expected total profit along the time horizon will be 290,802€, with an expected stock-out per period of 0 units and an expected final holding of 0 units per period.

Observing the behavior of the different optimal supply chain strategies, we induce some conclusions:

- The evaluated product is more sensitive to inventory than manufacturing costs: the increase in the production costs with 3D printing technologies (more expensive) is offset by the decrease in costs due to holding production (holding costs) and the backorders (stock-out costs).

- Installing 3D printing machines at car repair shops increases total profit by reducing stock levels and stock-out, although this strategy may not be able to supply all the demand in every scenario.
- The limitation of these processes in this system is the time $l$ each unit takes to be made; so a total number $N$ of machines working in parallel, with an average time $l/N$ by unit, is the determinant strategy factor.
- Given a demand rate $\lambda$, the number of 3D machines needed to supply all demand slightly varies with the value of the period lengths $t^P$, due to the approximations introduced in the demand generation procedure.

## 3D Printing Application in Toy Industry Customization

The next test case, proposed by Fundació CIM, corresponds to a toy company. Among many others, this company supplies collections of figurines from soccer teams, such as F.C. Barcelona or R.C.D. Espanyol. Each one is composed of about 20 items with a large range of demands.

Currently, the company designs the figurines at its headquarters near Barcelona, forecasts demand and outsources the orders to a Chinese manufacturing company. After constructing the molds and injecting some plastic to create the items, they are painted and delivered to Barcelona, a process that takes about 2 months. The proposed alternatives are to use a single color 3D printer (FDM-FFF) or a multicolor 3D printer (MJP-CJP). The main difference between these two technologies is that the second option substitutes injection and painting operations, but is more expensive than the first option (which only substitutes injection) in set-up, production costs and also in lead time.

The associated supply chain graph must contain:

**Market**

| | |
|---|---|
| $M_p$ | One market node for each figurine $p$ |

**Current**

| | |
|---|---|
| $I_p$ | One injection node (initial) for each figurine $p$ |
| $P_p^C$ | One painting node (distribution) for each figurine $p$ |

**Alternative FDM-FFF (single color)**

| | |
|---|---|
| $SC$ | One single color 3D printing node (initial) |
| $P_p^A$ | One painting node (distribution) located near $SC$, for each figurine $p$ |

**Alternative MJP-CJP (Multi-color)**

$MC$ One Multi-color 3D printing node (initial)

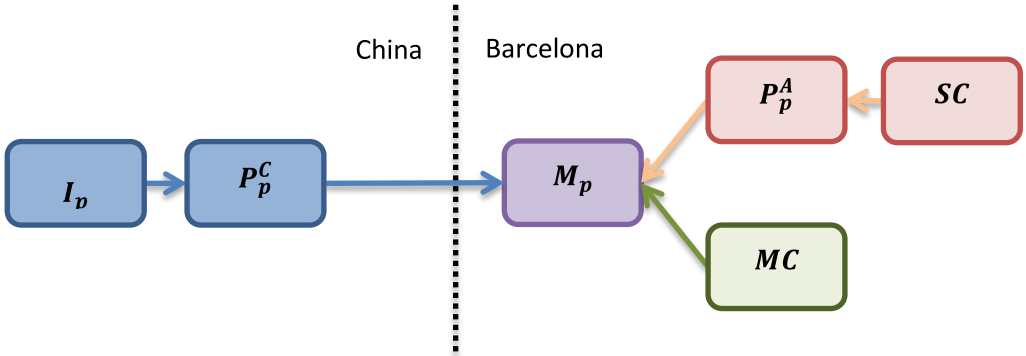


Figure 16: Supply chain graph of toy case

This model is made under the following assumptions:

1. The current process (blue) is composed, for each player $p$, by an injection node followed by a painting node located in China.
    a. Injection nodes $I_p$ are initial, they are characterized by the mold cost (set-up cost $f$), a selling price per unit (production cost $c$) and a lead time for producing a unit (production time $l$).
    b. Painting nodes $P_p^C$ are distribution (because of the relationship with their arcs), they are characterized by a painting and delivering unit cost (production cost $c$) and production unit lead time (production time $l$).
    c. We do not consider delivery time (fixed time $t$) in arcs $I_p \rightarrow P_p^C$, but we do in arcs $P_p^C \rightarrow M_p$.
2. Market (purple) is composed of a market node located in Barcelona for each player $p$.
    a. Each player $p$ has demand scenarios $d_s^p$ following a normal distribution $N(\mu_p, \sigma_p)$.
    b. There is a selling price and a stock-out cost for each player $p$, which we consider constant.
3. The alternative FDM-FFF process (red) is composed of only a single color printing node followed by a painting node for each player $p$, both located in Barcelona.
    a. Single color node $SC$ is initial. It is characterized by an installation cost (set-up cost $f$), a selling price per unit (production cost $c$), a lifetime ($q$) and a lead time for producing a unit (production time $l$).
    b. Painting nodes $P_p^A$ are distribution. We consider that the delivering unit cost offsets a higher painting cost, therefore we assume the same parameters as $P_p^C$.

c. We do not consider delivery time (fixed time $t$) in arcs $SC \to P_p^A$ but we do consider a small one in $P_p^A \to M_p$, assuming these operations are next to each other and close to the market.

4. The alternative MJP-CJP process (green) is composed of a single multi-color printing node, located in Barcelona.
   a. Multi-color node $MC$ is initial, it is characterized by an installation cost (set-up cost $f$), a selling price per unit (production cost $c$), a lifetime ($q$) and a lead time for producing a unit (production time $l$).
5. Both $SC$ and $MC$ are flexible technologies. So we assume its fixed cost $f$ is automatically amortized by its lifetime $q$, increasing their practical production cost $c$ to $\hat{c} = c + f/q$, while its practical fixed cost $f$ is set at 0.We present cost-time dataset for two cases, corresponding to two products with same costs but different demand:

We present the cost-time datasets for two cases, corresponding to two products with the same costs but different demand:

1. Production costs dataset:

| Operations | Node | Production costs $c_i$ $[€/unit]$ | Set-up costs $f_i$ $[€]$ | Life cycle $\bar{Q}_i$ $[unit]$ |
|---|---|---|---|---|
| **Injection $I$** | 1 | 0.15 | 6000 | – |
| **Painting $P^C$** | 2 | 0.05 | 500 | – |
| **Single color 3D $SC$** | 3 | 0.3 | 2600 | 16200 |
| **Painting $P^A$** | 4 | 0.03 | 1000 | – |
| **Multi-color 3D $MC$** | 5 | 0.5 | 90000 | 1450000 |
| **Market $M$** | 6 | – | – | – |

Table 18: Production costs of toy case

2. Holding costs dataset:

| Operations | Node | Holding costs $h_i$ $[€/unit]$ | Final holding costs $fh_i$ $[€/unit]$ | Set-up costs $z_i$ $[€]$ |
|---|---|---|---|---|
| **Injection $I$** | 1 | 0 | 0 | 1000 |
| **Painting $P^C$** | 2 | 0 | 0.05 | 1000 |
| **Single-color 3D $SC$** | 3 | 0 | 0 | 3000 |
| **Painting $P^A$** | 4 | 0 | 0.05 | 3000 |
| **Multi-color 3D $MC$** | 5 | 0 | 0 | 3000 |

| Market $M$ | 6 | 0.01 | 0.1 | 5000 |
|---|---|---|---|---|

Table 19: Holding costs of toy case

3. Variable lead times dataset:

| Node | Variable lead time $l_i$ $[h/unit]$ |
|---|---|
| **1** | 0.001 |
| **2** | 0.005 |
| **3** | 0.4 |
| **4** | 0.005 |
| **5** | 0.8 |
| **6** | – |

Table 20: Variable lead times of toy case

4. Fixed lead time dataset:

| Arc | Fixed lead time $t_i$ $[h]$ |
|---|---|
| $(1,2)$ | 0 |
| $(2,6)$ | 1400 |
| $(3,4)$ | 0 |
| $(4,6)$ | 1 |
| $(5,6)$ | 0 |

Table 21: Fixed lead times of toy case

5. Market dataset:

| Node | Selling price $p_i$ $[€/unit]$ | Stock-out costs $o_i$ $[€/unit]$ |
|---|---|---|
| **6** | 5 | 0.5 |

Table 22: Market parameters of toy case

We have generated 20 scenarios with 12 realizations each for both products presented. This instance has 21 1st stage binary variables, 14 1st stage nonnegative integer variables, 3120 2nd stage nonnegative integer variables, 2040 nonnegative continuous variables, 2766 equality constraints and 2442 inequality constraints. A relative MIP gap of 0.01 has been allowed for the following cases.

We will run realizations for two products with different demands over a horizon of 9 months, assuming maximum delivery time of one day (i.e., $t^P = 12\ h$) but allowing saturation rates of $\alpha = 0.5$.

## Product 1: Popular Team Star

This first product, corresponding to a popular veteran player, has a high demand expectation of $\mu_1 = 50{,}000\ u$. and a standard deviation of $\sigma_1 = \mu_1/3 \simeq 16{,}666\ u$. If we define the current injection technology as the only process available, the optimal solution is:

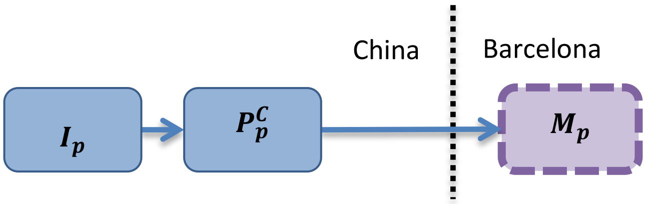


Figure 17: Current optimal of toy case 1

| Demand rate | Profit | Stock | Postponed production | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $\mathbf{90.24}\ \boldsymbol{u/p}$ | 202,669€ | $130\ u/p$ | – | $0.44\ u/p$ | $44\ u/p$ |

Table 23: Current optimal for toy case 1

In the current situation, an expected demand of 90.24 items/period is supplied speculatively by maintaining a holding quantity of 130 items/period at the decoupling point, which is market. This leads the stock-out rate to be the expected 0.44 items/period and an expected final holding of 44 items/ period.

Now, let us see what the optimal supply chain strategy is when the two 3D printing technologies (single and multicolor) are introduced as an alternative to the current injection process. The optimal solution of the $(OSCS)$ problem is:

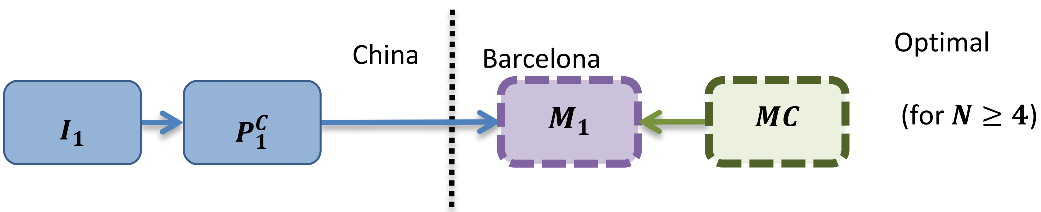


Figure 18: Optimal for toy case 1

| Demand rate | Profit | Stock | Postponed production | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $\mathbf{90.24}\ \boldsymbol{u/p}$ | 206,612€ | $91\ u/p$ | $11.54\ u/p$ | $0\ u/p$ | $16.14\ u/p$ |

Table 24: Optimal for toy case 1

The optimal solution keeps the current manufacturing process, but also adds 3D printing technologies to supply extra production in demand peaks when stock level decreases. The injection process manufactures in postponement 91 units/period, which are stocked at the decoupling point market, while the multi-color process manufactures an expected quantity of

11.54 units/ period in a postponed strategy that places the decoupling point at the multi-color operation.

This mixed strategy has a total profit of 206,612€, leads to a stock-out rate of zero and reduces the holding rate to one third. The single-color process is not installed in this solution.

### Product 2: New Player

The other product, corresponding to a young player, has an average demand expectation of $\mu_2 = 20{,}000\ u.$ and high standard deviation of $\sigma_2 = 0.6 \cdot \mu_2 \simeq 12{,}000\ u.$ The 20% lower tail, which is negative, has been computed as 0 (interpreted to mean that, in 20% of cases, the product will not have demand). Again, we can evaluate the current situation by defining the $(OSCS)$ problem with a single injection process. The solution thus obtained is:

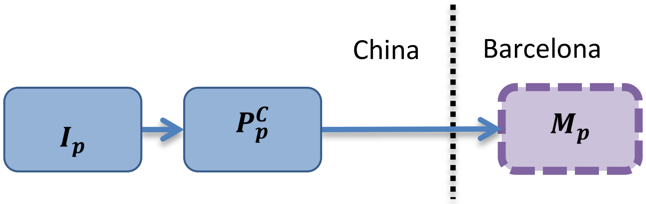


Figure 19: Current optimal of toy case 2

| Demand rate | Profit | Stock | Postponed production | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| **35.84 $u/p$** | 73,772€ | 70 $u/p$ | – | 0.35 $u/p$ | 34.61 $u/p$ |

Table 25: Current optimal of toy case 2

In the current situation, an expected demand of 35.84 items/period is supplied speculatively by maintaining a holding quantity of 70 items/period at the decoupling point, which is market. Total profit is 73,772€, leading to a stock-out rate of the expected 0.35 items/period and an expected final holding of 44 items/period.

Now, let us see what happens if we decide to install at least 3 3D printing machines for single-color and multi-color operations:

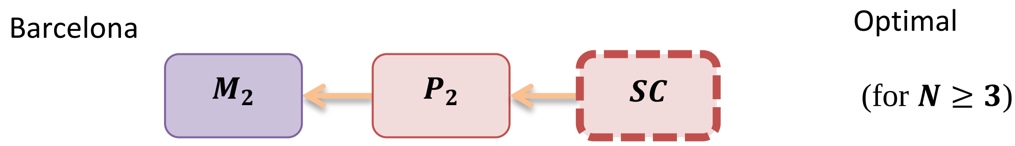


Figure 20: Optimal of toy case 2

| Demand rate | Profit | Stock | Postponed production | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $\mathbf{35.84}\ \boldsymbol{u/p}$ | 83,019€ | – | $35.73\ u/p$ | $0\ u/p$ | – |

Table 26: Optimal of toy case 2

The optimal solution for this product completely replaces the current process with single color 3D printing technology and a postponed strategy. Thus, holding and stock-out are reduced to 0. In this solution, a total profit of 83,019€ is reached by manufacturing an expected quantity of 35.73 units/period in the single color process. The decoupling point is then positioned at the single-color operation. In this solution, neither purchasing nor multi-color processes are installed.

From the obtained results we can conclude that:

- While products with steady demand (such as $\sigma_1 = 0.33 \cdot \mu_1$) run postponement strategies only during demand peaks, it is worth postponing all demand when uncertainty increases (such as $\sigma_2 = 0.6 \cdot \mu_2$).
- In both cases, some 3D printing machines are needed for obtaining time constraints. Although their production cost is high, it is compensated by reducing holding and stock-out costs.

## Craft Retail Company

Some studies have been done in the area of craft retail, seeking opportunities for 3D printing technologies in an effort to increase the degree of postponement as well as mass customization. Although we have studied a total of five products, we present here the study of just one selected product that allows us to show the presence of assembly nodes. We also show how we performed a graph reduction through the aggregation of nodes.

The selected product is a candle holder composed of three paws and a plate, all made of deburred metal. As with the previous cases, the goal is to choose the best manufacturing process from among two current possibilities (processes 1 and 2) and an alternative process (process 3) that is based on 3D printing manufacturing.

The associated supply chain graph must contain:

**Market**

| | |
|---|---|
| $M$ | One market node |
| | **Process 1** |
| $HB$ | One hot bending node (initial) for paw pieces. |
| $ST$ | One stamping node (initial) for plate pieces. |
| $D^1$ | One deburring node (distribution) for plate pieces |
| $A$ | One assembly node (assembly) for paws and plates. |
| | **Process 2** |
| $WC$ | One wax casting node (initial). |
| $D^2$ | One deburring node (distribution) for the whole item. |
| | **Process 3** |
| $3D$ | One metallic 3D printing node (initial) |
| $D^3$ | One deburring node (distribution) for the whole item. |
| $C$ | One covering node (distribution) for the whole item. |

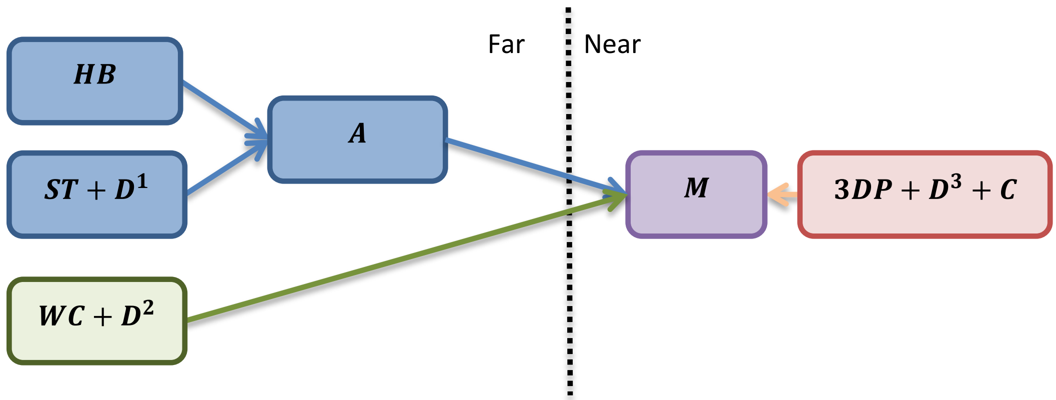


Figure 21: Supply chain of craft case

This model is made under the following assumptions:

1. Market $M$ (purple) is composed of a retail shop.

   a. We assume the given product to have an expected demand of $\mu = 18{,}000\ u$. with a high deviation of $\sigma = 0.5 \cdot \mu = 9{,}000\ u$.
   b. The selling price of this product is $p = 9€$, while the stock-out unit cost is set to $o = 3€$.
2. Process 1 (blue) is based on assembling paws to plates and delivering them to market.
   a. Paws are made in a hot bending process $HB$ (initial node), a quick and cheap operation.
   b. Plates are made in two steps: first they are created by a stamping process $ST$ (initial node), and then they are deburred at $D^1$ deburring process (distribution node). Apart from production cost and production lead time, the stamping operation has an initial set-up cost.
   c. Assembly is done at node $A$ from each three paws $HB \to A$ and one plate $ST + D^1 \to A$. Then, production is delivered to market $M$.
3. Process 2 (blue) is based on a wax casting process.
   a. Firstly, wax molds are built for creating each unit of complete product, which is costly and slow although highly customizable, at $WC$ (initial node).
   b. Immediately after, this complete product is deburred at $D^2$ (distribution node) and delivered to market $M$.
4. Process 3 (blue) is based on 3D printing technologies.
   a. Entire products are manufactured by metallic 3D printing techniques at operation $3D$, which is flexible but expensive and requires high initial investment.
   b. Then, production is deburred at $D^3$ and covered $C$ before delivering it to market.

In these process-selection problems, it is worth being careful in deciding what to define as a node. For instance, when two or more operations are such that

1. they are exclusively connected in a single path (i.e., no intermediate alternatives), and
2. all these operations are done consecutively, so we can neglect decisions of putting a decoupling point at an intermediate operation,

then we can just aggregate these operations into a single node and add together the set-up, production costs and production lead time, thus putting their lower values into the holding costs. That's what we did at process 1 coding node 2 as $ST + D^1$, node 3 as $WC + D^2$ and node 5 as $3DP + D^3 + C$, reducing the graph size to 6 nodes and 5 arcs.

1. Production costs dataset:

| Operations | Node | Production costs $c_i$ [€/unit] | Set-up costs $f_i$ [€] | Life cycle $\overline{Q}_i$ [unit] |
|---|---|---|---|---|
| $HB$ | 1 | 0.35 | 0 | – |
| $ST + D^1$ | 2 | 0.32 | 2250 | – |
| $WC + D^2$ | 3 | 5.27 | 15000 | – |
| $A$ | 4 | 0.5 | 0 | – |
| $3DP + D^3 + C$ | 5 | 7.37 | – | – |
| $M$ | 6 | – | – | – |

Table 27: Production costs of craft case

2. Holding costs dataset for non-assembly nodes:

| Operations | Node | Holding costs $h_i$ [€/unit] | Final holding costs $fh_i$ [€/unit] | Set-up costs $z_i$ [€] |
|---|---|---|---|---|
| $HB$ | 1 | 0.05 | 1 | 1000 |
| $ST + D^1$ | 2 | 0.05 | 1.5 | 1000 |
| $WC + D^2$ | 3 | 0.05 | 1.5 | 1000 |
| $3DP + D^3 + C$ | 5 | 0.05 | 2 | 1000 |
| $M$ | 6 | 0.1 | 2.5 | 2000 |

Table 28: Non assembly holding costs of craft case

3. Holding costs dataset for assembly nodes:

| Flow | Arc | Holding costs $ah_{ij}$ [€/unit] | Final holding costs $fah_{ij}$ [€/unit] | Piece ratio $r_{ij}$ [unit] |
|---|---|---|---|---|
| $HB \rightarrow A$ | (1,4) | 0.05 | 1 | 3 |
| $ST + D^1 \rightarrow A$ | (2,4) | 0.05 | 1.5 | 1 |

Table 29: Assembly holding costs of craft case

4. Variable lead times dataset:

| Node | Variable lead time $l_i$ [h/unit] |
|---|---|

5. Fixed lead time dataset:

| Arc | Fixed lead time $t_i$ [h] |
|---|---|

| | |
|---|---|
| **1** | 0.012 |
| **2** | 0.013 |
| **3** | 0.458 |
| **4** | 0.008 |
| **5** | 0.052 |
| **6** | – |

Table 30: Variable lead times of craft case

| | |
|---|---|
| $(\mathbf{1},\mathbf{4})$ | 0 |
| $(\mathbf{2},\mathbf{4})$ | 0 |
| $(\mathbf{3},\mathbf{6})$ | 720 |
| $(\mathbf{4},\mathbf{6})$ | 720 |
| $(\mathbf{5},\mathbf{6})$ | 4 |

Table 31: Fixed lead time of craft case

6. Market dataset:

| Node | Selling price $p_i$ [€/$unit$] | Stock-out costs $o_i$ [€/$unit$] |
|---|---|---|
| **6** | 9 | 3 |

Table 32: Market parameters of craft case

We have generated a sample of 20 scenarios with 12 period realizations each, following a normal distribution with parameters $N(\mu = 18000, \sigma = 900)$. This instance has 21 binary 1st stage variables, 16 1st stage nonnegative integer variables, 3360 2nd stage nonnegative integer variables, 2060 2nd stage nonnegative continuous variables, 3267 equality constraints and 2443 inequality constraints. The following cases have been resolved while allowing a relative MIP gap of 0.01.

We have scanned solutions for all 0-to-1 values of $\alpha$, any number of 3D printing machines and with delivery times from one day to one month (i.e., $t^P = 12\ h$ to $t^P = 360\ h$). In all these cases, the optimal solution was to speculate and manufacture from process 1. We next show some statistics on the instance with $\alpha = 1$, $N = 100$ and $t^P = 84\ h$ (delivery time of one week):

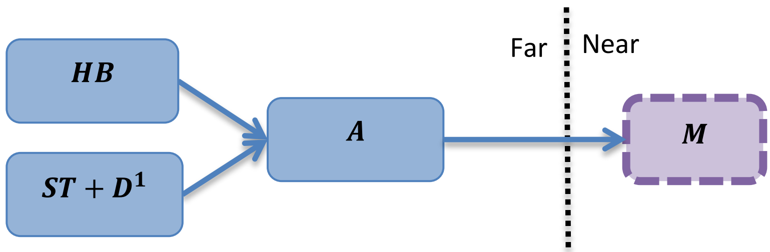


Figure 22: Optimal of craft case

| Demand rate | Profit | Stock | Postponed production | Stock-out rate | Holding rate |
|---|---|---|---|---|---|
| $\mathbf{166.03}\ \boldsymbol{u/p}$ | 59,074€ | $231\ u/p$ | – | $12.21\ u/p$ | $85.23\ u/p$ |

Table 33: Optimal of craft case

The optimal of this case is to manufacture speculatively process 1, putting a decoupling point at market. This solution supplies a half-weekly demand rate of 166 units by maintaining a stock level of 231 units/period. The total profit after the time horizon is 59,074€, with an expected stock-out rate of 12.21 units/period, and an expected final holding of 85.23 units.

The main conclusion of this case is that the incremental cost of introducing 3D printing techniques does not compensate for the benefits of reducing stocks and backordering; unlike before, time restrictions are not active constraints in the 3D printing production.

## Application for Multiple Products with Geographical Configuration

All the previous test cases are real company problems with real data, but their supply chain modeling and resolution have not illustrated the scope of the model presented. To demonstrate the potential of our model, we present this last test case, in which we solve an $(OSCS)$ problem with a family of products that have a common *Core* and differentiated *Features,* where each part can be manufactured or assembled in different geographical places and with different speculation/postponement strategies.

Assume we have a family of two products with the same *Core* and differentiated features, *Feat1* and *Feat2*; and these three parts can be manufactured and assembled in two geographical places, let us say *Far* and *Near*.

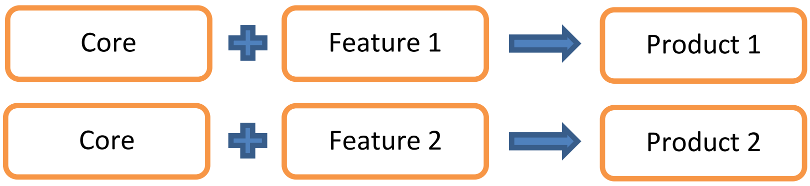


Figure 23: Pieces of products of fourth case

Given a time-cost dataset and some information about demand, we face a problem with multiple key decisions, which are:

- The **geographical configuration** of the supply chain.
- The **postponement/speculation** strategy of that configuration.

**Market**

$M_1$ One market node for product 1

$M_2$ One market node for product 2

**Geographical Zone Far**

$C^F$ One core manufacturing node (initial) located in Far

$F_1^F$ One feature1 manufacturing node (initial) located in Far

$F_2^F$ One feature1 manufacturing node (initial) located in Far

$A_1^F$ One assembly node of product 1 located in Far

$A_2^F$ One assembly node of product 2 located in Far

**Geographical Zone Near**

$C^N$ One core manufacturing node (initial) located in Near

$F_1^N$ One feature1 manufacturing node (initial) located in Near

$F_2^N$ One feature1 manufacturing node (initial) located in Near

$A_1^N$ One assembly node of product 1 located in Near

$A_2^N$ One assembly node of product 2 located in Near

**Artificial flow nodes**

$D_C$ One artificial node (distribution) for channel core production

$D_{F1}$ One artificial node (distribution) for channel feat1 production

$D_{F2}$ One artificial node (distribution) for channel feat2 production

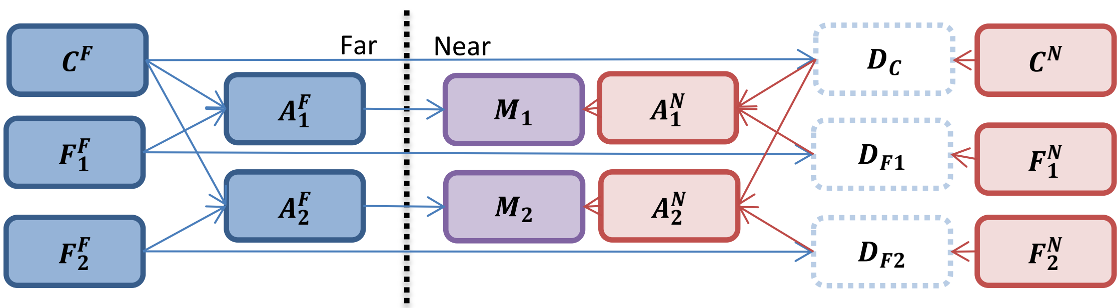


Figure 24: Supply chain for fourth case

We design this model under the following assumptions:

1. There are two market nodes: $M_1$ for product 1 and $M_2$ for product 2, located *Near*.
2. There are manufacturing operations (initial nodes) that generate all parts, located both *Far* and *Near*:
   a. *Core* can be manufactured *Far* $C^F$ or *Near* $C^N$.
   b. Feature 1 can be manufactured Far $F_1^F$ or Near $F_1^N$.
   c. Feature 2 can be manufactured Far $F_2^F$ or Near $F_2^N$.
3. There are assembly operations that generate products, located both *Far* and *Near*:
   a. We declare two assembly nodes as the assembly operation located *Far,* $A_1^F$ for product 1 and $A_2^F$ for product 2. Only production manufactured *Far* can be assembled *Far,* otherwise it can be delivered to *Near.*
   b. We declare two assembly nodes as the assembly operation located *Near*: $A_1^N$ for product 1 and $A_2^N$ for product 2. These nodes can assemble production manufactured both *Far* and *Near*.
4. Because assembly nodes $A_1^N$ and $A_2^N$ can take the same part from different operations, it is necessary to include artificial nodes to channel same pieces to single arcs:
   a. Artificial operation $D_C$ (production node) adds the *Core* production made in $C^F$ sent to *Near* and the *Core* quantity made in $C^N$, then channels them to $A_1^N$ and $A_2^N$.
   b. Artificial operation $D_{F1}$ (production node) adds the *Feature 1* production made in $F_1^F$ sent to *Near* and the *Feature 1* production made in $F_1^N$, then channels them to $A_1^N$.
   c. Artificial operation $D_{F2}$ (production node) adds the *Feature 2* production made in $F_2^F$ sent to *Near* and the *Feature 2* production made in $F_2^N$, then channels them to $A_2^N$.

We present the cost-time dataset of this case in a set of tables:

1. Production costs dataset:

| Operations | Node | Production costs $c_i$ $[€/unit]$ | Set-up costs $f_i$ $[€]$ | Life cycle $\bar{Q}_i$ $[unit]$ |
|---|---|---|---|---|
| **Core Far** | 1 | 3 | 1000 | – |
| **Feature 1 Far** | 2 | 2 | 1000 | – |
| **Feature 2 Far** | 3 | 2 | 1000 | – |
| **Assembly 1 Far** | 4 | 1 | 1000 | – |
| **Assembly 2 Far** | 5 | 1 | 1000 | – |
| **Core Near** | 6 | 3 | 3000 | – |
| **Feature 1 Near** | 7 | 2 | 3000 | – |

| | | | | |
|---|---|---|---|---|
| **Feature 2 Near** | 8 | 2 | 3000 | – |
| **Assembly 1 Far** | 9 | 1 | 3000 | – |
| **Assembly 2 Near** | 10 | 1 | 3000 | – |
| **Market 1** | 11 | – | – | – |
| **Market 2** | 12 | – | – | – |
| **Artificial flow** | 13 | 0 | 0 | – |
| **Artificial flow** | 14 | 0 | 0 | – |
| **Artificial flow** | 15 | 0 | 0 | – |

Table 34: Production costs of fourth case

2. Holding costs dataset for non-assembly nodes:

| Operations | Node | Holding costs $h_i$ [€/unit] | Final holding costs $fh_i$ [€/unit] | Set-up costs $z_i$ [€] |
|---|---|---|---|---|
| **Core Far** | 1 | 0.1 | 1 | 1000 |
| **Feature 1 Far** | 2 | 0.05 | 0.3 | 1000 |
| **Feature 2 Far** | 3 | 0.05 | 0.3 | 1000 |
| **Assembly 1 Far** | 4 | – | – | 2000 |
| **Assembly 2 Far** | 5 | – | – | 2000 |
| **Core Near** | 6 | 0.2 | 2 | 6000 |
| **Feature 1 Near** | 7 | 0.1 | 0.6 | 6000 |
| **Feature 2 Near** | 8 | 0.1 | 0.6 | 6000 |
| **Assembly 1 Far** | 9 | – | – | 8000 |
| **Assembly 2 Near** | 10 | – | – | 8000 |
| **Market 1** | 11 | 0.3 | 3 | 10000 |
| **Market 2** | 12 | 0.3 | 3 | 10000 |
| **Artificial flow** | 13 | – | – | – |
| **Artificial flow** | 14 | – | – | – |
| **Artificial flow** | 15 | – | – | – |

Table 35: Non-assembly holding costs of fourth case

3. Holding costs dataset for assembly nodes:

| Flow | Arc | Holding costs $ah_{ij}$ $[€/unit]$ | Final holding costs $fah_{ij}$ $[€/unit]$ | Piece ratio $r_{ij}$ $[unit]$ |
|---|---|---|---|---|
| **Core -> As 1 Far** | (1,4) | 0.1 | 1 | 1 |
| **Core -> As 2 Far** | (1,5) | 0.1 | 1 | 1 |
| **Feat1 -> As 1 Far** | (2,4) | 0.05 | 0.3 | 1 |
| **Feat2 -> As 2 Far** | (3,5) | 0.05 | 0.3 | 1 |
| **Core -> As 1 Near** | (13,9) | 0.2 | 2 | 1 |
| **Core -> As 2 Near** | (13,10) | 0.2 | 2 | 1 |
| **Feat1 -> As 1 Near** | (14,9) | 0.1 | 0.6 | 1 |
| **Feat2 -> As 2 Near** | (15,10) | 0.1 | 0.6 | 1 |

Table 36: Assembly holding costs of fourth case

4. Variable lead time dataset:

| Node | Variable lead time $l_i$ $[h/unit]$ |
|---|---|
| **1** | 0.2 |
| **2** | 0.05 |
| **3** | 0.05 |
| **4** | 0.05 |
| **5** | 0.05 |
| **6** | 0.3 |
| **7** | 0.1 |
| **8** | 0.1 |
| **9** | 0.1 |
| **10** | 0.1 |
| **11** | – |
| **12** | – |
| **13** | 0 |
| **14** | 0 |
| **15** | 0 |

Table 37: Variable lead times of fourth case

5. Fixed lead time dataset:

| Arc | Fixed lead time $t_i$ $[h]$ |
|---|---|
| $(\mathbf{1},\mathbf{4})$ | 0 |
| $(\mathbf{1},\mathbf{5})$ | 0 |
| $(\mathbf{1},\mathbf{13})$ | 168 |
| $(\mathbf{2},\mathbf{4})$ | 0 |
| $(\mathbf{3},\mathbf{5})$ | 0 |
| $(\mathbf{2},\mathbf{14})$ | 168 |
| $(\mathbf{3},\mathbf{15})$ | 168 |
| $(\mathbf{4},\mathbf{11})$ | 72 |
| $(\mathbf{5},\mathbf{12})$ | 72 |
| $(\mathbf{6},\mathbf{13})$ | 2 |
| $(\mathbf{7},\mathbf{14})$ | 2 |
| $(\mathbf{8},\mathbf{15})$ | 2 |
| $(\mathbf{13},\mathbf{9})$ | 0 |
| $(\mathbf{13},\mathbf{10})$ | 0 |
| $(\mathbf{14},\mathbf{9})$ | 0 |
| $(\mathbf{15},\mathbf{10})$ | 0 |
| $(\mathbf{9},\mathbf{11})$ | 0 |
| $(\mathbf{10},\mathbf{12})$ | 0 |

Table 38: Fixed lead times of fourth case

We have generated a sample of 12 scenarios with 5 period realizations each. This instance has 64 binary 1st stage variables, 43 1st stage nonnegative integer variables, 2,460 2nd stage nonnegative integer variables, 1,368 2nd stage nonnegative continuous variables, 1,987 equality constraints and 2,308 inequality constraints. The following cases have been resolved while allowing a relative MIP gap of 0.01.

## Case 1: Complementary Products

The first case studies a pair of complementary products, i.e., products whose demand has positive correlation. Both products follow a normal distribution with the respective parameters $\mu_1 = 9{,}000, \sigma_1 = 5{,}000$ and $\mu_2 = 11{,}000, \sigma_2 = 5{,}000$, and a correlation of $\rho_{12} = 0.5$.

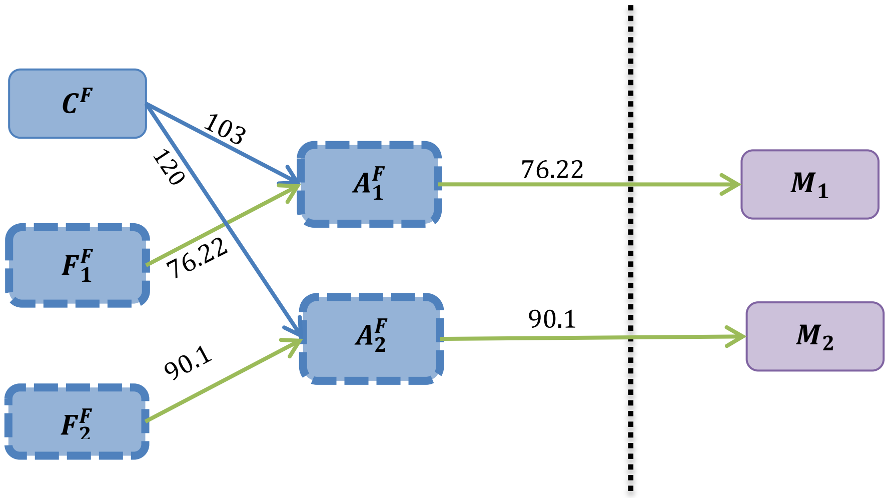


Figure 25: Optimal of fourth case 1

In the above figure, the optimal is shown by drawing blue speculative lines and green postponed ones. On the blue ones, the number corresponds to the speculative production per period; while on the green ones, it corresponds to the expected postponed production per period.

This solution, which has a total profit of 16,916€, installs all five operations located *Far* by working in a mixed postponement/speculation strategy. This mixed strategy consists of installing the following decoupling points:

| Decoupling point | Stock level per period | Expected released production per period | Expected holding production per period |
|---|---|---|---|
| $F_1^F$ | 103 u/p | $76.22\ u/p \rightarrow A_1^F$ | 26.78 u/p |
| $F_2^F$ | 120 u/p | $90.1\ u/p \rightarrow A_2^F$ | 29.9 u/p |
| $A_1^F$ | $C^F \rightarrow 103\ u/p$ | $76.22\ u/p \rightarrow M_1$ | $C^F \rightarrow$ 26.78 u/p |
| $A_2^F$ | $C^F \rightarrow 120\ u/p$ | $90.1\ u/p \rightarrow M_2$ | $C^F \rightarrow$ 29.9 u/p |

Table 39: Decoupling points of fourth case 1

Core manufacturing operation $C^F$ manufactures speculatively in order to maintain stock levels of 103 units/period of piece *Core* at $A_1^F$ and 120 units/period at $A_2^F$. The stock level of 103 units/period for the postponed operation $F_1^F$ means that it should be ready to manufacture up to 103 units/period of piece *Feat1*. The same interpretation applies to the stock level of 120 units/period of piece *Feat2* of the postponed operation $F_2^F$.

When demand is disclosed, $\boldsymbol{F_1^F}$ manufactures an expected quantity of 76.22 units/period of piece *Feat1* that are moved to $\boldsymbol{A_1^F}$, assembled to *Core* and delivered to $\boldsymbol{M_1}$; while $\boldsymbol{F_2^F}$ manufactures an expected quantity of 90.1 units/period that are moved to $\boldsymbol{A_2^F}$, assembled to *Core* and delivered to $\boldsymbol{M_2}$. Finally, $\boldsymbol{F_1^F}$ will store an expected quantity of 26.78 *Feat1* pieces, $\boldsymbol{A_1^F}$ an expected quantity of 26.78 *Core* pieces, $\boldsymbol{F_2^F}$ an expected quantity of 29.9 *Feat2* pieces, and $\boldsymbol{A_2^F}$ an expected quantity of 29.9 *Core* pieces.

| Market | Expected demand per period | Expected sales per period | Expected stock-out per period |
|---|---|---|---|
| $\boldsymbol{M_1}$ | 89.88 u/p | 76.22 $u/p$ | 13.66 u/p |
| $\boldsymbol{M_2}$ | 100.53 u/p | 90.1 $u/p$ | 10.42 u/p |

Table 40: Markets of fourth case 1

## Case 2: Substitutable Products

The second case studies a pair of substitutable products, i.e., products whose demand has negative correlation. Both products follow a normal distribution with the respective parameters $\mu_1 = 9{,}000, \sigma_1 = 5{,}000$ and $\mu_2 = 11{,}000, \sigma_2 = 5{,}000$, and a correlation of $\rho_{12} = -0.5$.

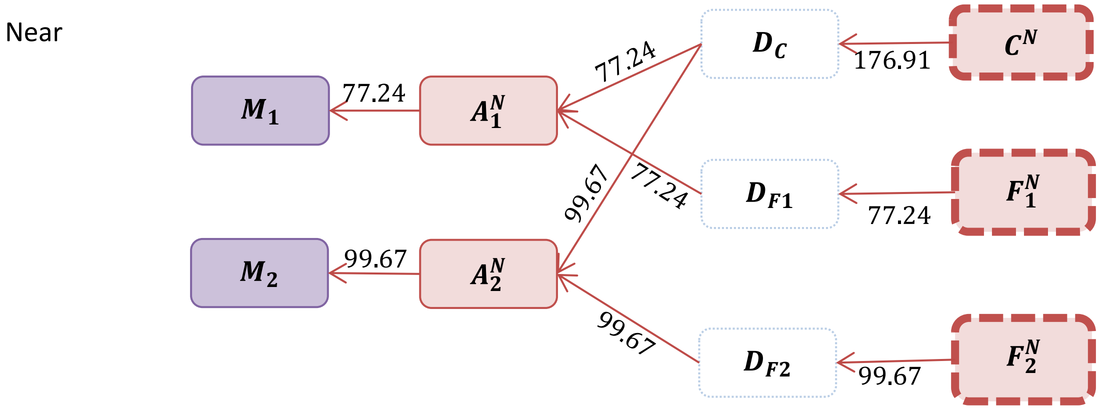


Figure 26: Optimal of fourth case 2

This solution, which has a total profit of 14,954€, installs all five operations located *Near* by working in a pure postponement strategy. This pure strategy consists of installing the following decoupling points:

| Decoupling point | Stock level per period | Expected released production per period | Expected holding production per period |
|---|---|---|---|
| $\boldsymbol{C^N}$ | $226\ u/p$ | $77.24\ u/p \rightarrow \boldsymbol{A_1^N}$<br>$99.67\ u/p \rightarrow \boldsymbol{A_2^N}$ | $49.08\ u/p$ |
| $\boldsymbol{F_1^N}$ | $140\ u/p$ | $77.24\ u/p \rightarrow \boldsymbol{A_1^N}$ | $62.76\ u/p$ |
| $\boldsymbol{F_2^N}$ | $151\ u/p$ | $99.67\ u/p \rightarrow \boldsymbol{A_2^N}$ | $51.33\ u/p$ |

Table 41: Decoupling points of fourth case 2

The stock level of 226 units/period for the *Core* manufacturing operation $\boldsymbol{C^N}$ means that it should be ready to manufacture up to 226 units/period in postponement. The same interpretation applies to the stock level of 140 units/period of the *Feat1* manufacturing operation $\boldsymbol{F_1^N}$, and to the stock level of 151 units/period of the *Feat1* manufacturing operation $\boldsymbol{F_2^N}$.

When demand is disclosed, $\boldsymbol{C^N}$ manufactures an expected quantity of 176.91 units/period of *Core*, which are moved to $\boldsymbol{A_1^N}$ ($77.24\ u/p$) and $\boldsymbol{A_2^N}$ ($99.67\ u/p$); $\boldsymbol{F_1^N}$ manufactures an expected quantity of 77.24 units/period, which are moved to $\boldsymbol{A_1^N}$; and $\boldsymbol{F_2^N}$ manufactures an expected quantity of 99.67 units/period, which are moved to $\boldsymbol{A_2^N}$. Then, $\boldsymbol{A_1^N}$ and $\boldsymbol{A_2^N}$ deliver all production to markets $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$. Finally, $\boldsymbol{C^N}$ will store an expected quantity of 49.08 *Core* pieces, $F_1^N$ an expected quantity of 62.76 *Feat1* pieces, and $\boldsymbol{F_2^F}$ an expected quantity of 99.67 *Feat2* pieces.

| Market | Expected demand per period | Expected sales per period | Expected stock-out per period |
|---|---|---|---|
| $\boldsymbol{M_1}$ | 78.05 u/p | $77.24\ u/p$ | 0.8 u/p |
| $\boldsymbol{M_2}$ | 100.25 u/p | $99.67\ u/p$ | 0.58 u/p |

Table 42: Markets of fourth case 2

Our conclusion is that the main strategic differences between the cases are the operations installed: those located *Far* in Case 1 and those located *Near* in Case 2.

In Case 1, the fixed time to deliver production to markets from assembly operations is 72 of the 84 hours in each time period. For this reason, there is not enough time to manufacture *Core* pieces in postponement, so they are speculated and buffered at the assembly operations, while manufacturing *Feat1* and *Feat2* is a bit quicker and can be done in postponement.

In Case 2, the fixed fixed time to deliver production to markets from assembly operations is $2$ of the $84$ hours in each time period. Now there is enough time to postpone the entire chain and move production to market in the planned time.

The standard deviation of demand for both products (that is, the demand for *Core*) is about $\Sigma \simeq 9{,}000\, u.$ in Case $1$ (correlation $\rho = 0.5$) while it decreases to $\Sigma \simeq 5{,}000\, u.$ in Case $2$ (correlation $\rho = -0.5$). This is a counterintuitive result of the manufacturing strategy of *Core* operations: it is natural to think there would be a higher degree of postponemet for higher deviation in demand, which would lead us to postpone *Core* manufacturing in the first case and speculate on it in the second. However, manufacturing operations are different for each case, so intuition may fail.

# 7. Conclusions and further research

This thesis proposes a new stochastic programming model for dealing with the strategic design of any supply chain, making it possible to decide on the optimal manufacturing processes as a set of supply chain operations. These include manufacturing, assembly, stocking, distribution and delivering to markets with stochastic demand. For each one of the selected processes, the model provides the optimal degree of postponement, i.e., those points that separate the forecast-driven production (speculation) from the order-driven production (postponement) in a flow of goods. These separation points are known as customer order decoupling points.

The motivation for this work was to find an answer to the question "*When should an enterprise implement certain strategic supply chain models that use 3D Printing?*" The solution was sought in collaboration on a research project between the UPC and Accenture. Early in the study of this problem, it became evident that the analyses could not be restricted to the technological specificities of 3D printers but instead had to adopt a broader scope by taking into consideration that 3D printers are one stage in a sequence of operations along the entire supply chain. The development of this idea gave birth to a general methodology, the $(OSCS)$ model, which allows choosing the best from among a given portfolio of technologies, whether they be 3D printing or any other. The development of this methodology was deployed progressively by means of the following steps:

1. An extensive review of the existing bibliography on analytical methods, in order to evaluate supply chain strategies (speculation/postponement) and manufacturing with 3D printing.
2. The compilation and definition of a series of test cases from Accenture's clients and FundacióCIM, specifically those for whom 3D printing seemed to be a promising alternative manufacturing technology for their supply chains.
3. The development of a stochastic programming optimization model for finding the Optimal Supply Chain Strategy, the so-called $(OSCS)$.
4. The computational implementation of the $(OSCS)$ model in the AMPL language.
5. Through analysis of the numerical results, verification that the $(OSCS)$ model is a valid tool for assessing how to introduce 3D printing and postponement into the supply strategies of the test cases defined so far.

The main conclusions drawn from this project are:

1. That stochastic programming has proved to be an appropriate tool for modeling strategic supply chain decisions to decide between postponement/speculative manufacturing with stochastic demand.
2. That the methodology to assess the convenience of the introduction of 3D printing technologies in manufacturing processes can be generalized to determine the best technology among a portfolio of alternative technologies.
3. The application of the $(OSCS)$ model to the set of test cases reveals that
    a. Postponement strategies are preferable when the evaluated product is more sensitive to inventory than manufacturing costs. At these cases, the increase in the production costs of selecting technologies able to manufacture in postponement is offset by the decrease in costs due to holding production and stock-out.
    b. 3D printing technologies are often preferable for manufacturing in postponement, but a single machine may not manufacture some quantities in a given time period. When time constraints limit 3D printing production quantity, a critical analysis on the number of 3D printing machines working in parallel illustrates a trade-off between initial investment in 3D printing technologies and the amount of production manufactured in a postponement strategy.
    c. High degrees of uncertainty in demand also facilitate the introduction of postponement. While products with steady demand may run postponement strategies only during demand peaks, it is worth postponing all demand when uncertainty increases.

## Contributions

There are three main contributions of this thesis to the existing literature:

1. A flexible network configuration of a global supply chain that considers generic manufacturing, assembling, distribution and selling operations. The flexibility allows to hold in a single optimization problem the formulation of a wide range of classical and new supply chain strategy problems, such as process selection, postponement degree, location and product differentiation.
2. A new approach to the treatment of the classical speculation/postponement dilemma through stochastic programming based on the association of the speculative decision to the first stage variables and the postponement decision to the recourse variables.
3. A novel treatment of uncertainty in the demand, one which considers randomness between time intervals in addition to the stochasticity in the total demand quantity. This makes it possible to take into account an approximation of the customer's waiting time without the need to model a multi-period program or an explicit queuing system.

## Further Research

1. From the point of view of the mathematical modeling, the most natural advance in the optimization framework would concern a multi-objective analysis of this model. Aside from the current profit function, some other quantities can be considered for optimization, such as delivery time, saturation rate and the number of machines working in parallel.
2. From the point of view of the practical use of the $(OSCS)$ model as a decision-making tool, one useful advance would be to develop user-friendly software for data visualization. This would help companies introduce their cases and graphically represent both strategy decisions and a comprehensible visualization of some 2nd stage results concerning sales, production, holding, stock-out and lead times.
3. From the point of view of the optimization algorithms, some specialized decomposition techniques for large scale optimization could be necessary to deal with instances larger than the one presented in this thesis. The structure of the second stage sub-problems could allow analytical methods to find either optimum or good enough solutions in more efficient ways than the general purpose optimization software used so far.

# 8.Notation

| Variable | Symbol | Stage | Domain | Size |
|---|---|---|---|---|
| **Selected Arc** | $X^{ij}$ | $1^{st}$ | $\{0,1\}$ | $\|L\|$ |
| **Speculation/Postponement Strategy** | $Y^{ij}$ | $1^{st}$ | $\{0,1\}$ | $\|L\|$ |
| **Decoupling Point** | $Z^{j}$ | $1^{st}$ | $\{0,1\}$ | $\|N\|$ |
| **Selected Operation** | $W^{j}$ | $1^{st}$ | $\{0,1\}$ | $\|N \setminus M\|$ |
| **Speculative Production** | $P_0^{ij}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|L\|$ |
| **Initial Production** | $K^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|I\|$ |
| **Initial Holding** | $H^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|N \setminus A\|$ |
| **Initial Assembly Holding** | $AH^{j}$ | $1^{st}$ | $\mathbb{Z}_0^+$ | $\|D(A)\|$ |
| **Postponed Production** | $P_{sq}^{ij}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|L \times \Omega \times Q_s\|$ |
| **Released Production** | $R_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|(N \setminus A) \times \Omega \times Q_s\|$ |
| **Released Assembly Production** | $AR_{sq}^{ij}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|D(A) \times \Omega \times Q_s\|$ |
| **Sales** | $S_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Stock-Out** | $O_{sq}^{j}$ | $2^{nd}$ | $\mathbb{Z}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Postponement lead time** | $T_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|N \times \Omega \times Q_s\|$ |
| **Idle time** | $U_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Saturation time** | $V_{sq}^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|M \times \Omega \times Q_s\|$ |
| **Final Holding** | $F_s^{j}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|(N \setminus A) \times \Omega\|$ |
| **Final Assembly Holding** | $AF_s^{ij}$ | $2^{nd}$ | $\mathbb{R}_0^+$ | $\|D(A) \times \Omega\|$ |

Table 43: Table of variables

| Set | Symbol |
|---|---|
| Nodes | $N$ |
| Initial Nodes | $I$ |
| Assembly Nodes | $A$ |
| Production Nodes | $D$ |
| Market Nodes | $M$ |
| Lines | $L$ |
| Scenarios | $\Omega$ |
| Realizations of $s$ | $Q_s$ |

Table 44: Table of Sets

| Parameter | Symbol | Dim. | Domain |
|---|---|---|---|
| Cardinality of $N$ | $n$ | $-$ | $-$ |
| Cardinality of $\Omega$ | $s_{set}$ | $-$ | $-$ |
| Cardinality of $Q_s$ | $q_{set}$ | $-$ | $-$ |
| Number of time periods | $n^P$ | $-$ | $-$ |
| Length of time periods | $t^P$ | $h$ | $-$ |
| Saturation rate | $\alpha$ | $-$ | $-$ |
| Probability of scenarios | $\omega_s$ | $-$ | $\Omega$ |
| Probability of realizations | $\pi_{sq}$ | $-$ | $\Omega \times Q_s$ |
| Total demand | $d_s^j$ | $u$ | $M \times \Omega$ |
| Demand of realization $(s, q)$ | ${d^P}_{sq}^j$ | $u/p$ | $M \times \Omega \times Q_s$ |
| Operation production cost | $p_j$ | $€/u$ | $N \setminus M$ |
| Operation set-up cost | $f_j$ | $€$ | $N \setminus M$ |
| Operation lifetime | $q_j$ | $u$ | $N \setminus M$ |
| Initial holding cost | $h_j$ | $€/u$ | $N \setminus A$ |
| Final holding cost | $fh_j$ | $€/u$ | $N \setminus A$ |
| Initial assembly holding cost | $ah_{ij}$ | $€/u$ | $D(A)$ |
| Final assembly holding cost | $fah_{ij}$ | $€/u$ | $D(A)$ |
| Ratio of pieces | $r_{ij}$ | $u$ | $D(A)$ |
| Buffer set-up cost | $z_j$ | $€$ | $N$ |
| Stock-out cost | $o_j$ | $€/u$ | $M$ |
| Selling price | $p_j$ | $€/u$ | $M$ |
| Production time | $l_i$ | $h/u$ | $N \setminus M$ |
| Distribution time | $t_{ij}$ | $h$ | $L$ |

Table 45: Table of parameters

# 9. Table of Figures and Tables

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